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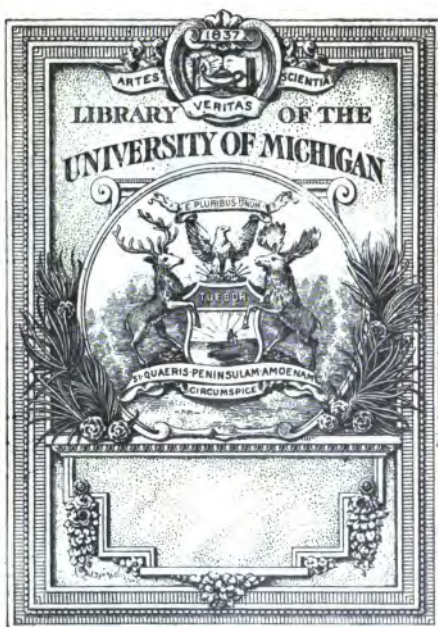
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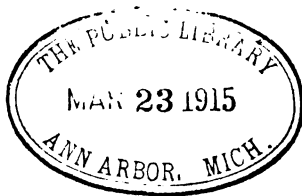
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ICE-MAKING MACHINES:

THE THEORY OF THE ACTION OF THE VARIOUS
FORMS OF COLD-PRODUCING OR
SO-CALLED ICE MACHINES

TRANSLATED FROM THE FRENCH OF

M. LEDOUX,

Ingenieur des Mines.

REVISED AND TRANSFORMED TO ENGLISH UNITS

BY

J. E. DENTON, D. S. JACOBUS

AND

A. RIESENBERGER.

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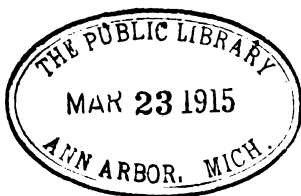
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Class.....

Book.....

Accession.....

08-19-2117



PREFACE.

This masterly essay, by M. Ledoux.

ERRATA.

On page xli for five tons capacity read eight tons capacity.

On page xlii for three pounds of ice-melting capacity read seven pounds of ice-melting capacity.

chemistry have found it to their interest to be guided by the numerical results of M. Ledoux; and theoretical writers are indebted to him for the general method, by which the latent heat and density of ammonia and sulphur dioxide were arrived at theoretically, at a time when

iii

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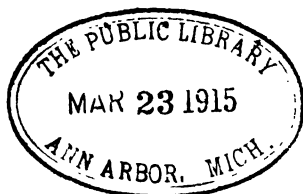
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Class.....

Book.....

Accession.....

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PREFACE.

This masterly essay, by M. Ledoux, giving a thermodynamic discussion of the various fluids used for the artificial production of cold, was published in the "Annales des Mines" about twelve years ago. The essay, as translated and published in Van Nostrand's Science Series, has proved a valuable source of information to those who thoroughly investigated its contents.

Leading makers of refrigerating machinery have found it to their interest to be guided by the numerical results of M. Ledoux; and theoretical writers are indebted to him for the general method, by which the latent heat and density of ammonia and sulphur dioxide were arrived at theoretically, at a time when

iii

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no experimental determinations of these important constants were known.

The mathematical treatment of the subject was very elaborate and well illustrated by numerical examples showing the relative value of air, ether, sulphur dioxide, and ammonia, as used in compression and absorption systems, but as all figures in the translation remained in metric units, the value of the work has been appreciated by few engineers.

The present revision of the Van Nostrand translation has been undertaken to make Ledoux's work more available to American students. All figures have been transformed to English units, all results expressed in the units commonly used in American practice, and many typographical errors eliminated from the text.

Intermediate steps not fully indicated in the original have been supplied in the form of foot-notes. The latter have also been liberally introduced to emphasize important points of the text, and to call attention to modifications of the

author's views, suggested by the most recent practice and experiments.

Some of the most intricate analysis regarding air is unnecessary in calculating the tables of performance. It, however, is excellent material for teachers of thermodynamics, seeking exercises in applied mathematics for their students, as are, also, the differential equations used to establish the adiabatic relations of superheated vapors of ammonia and sulphur dioxide.

Tables of the properties of carbonic acid and of the "Pictet fluid," as far as yet known, have been added, and the numerical examples have been made to include carbonic acid, by the use of the formulæ published by Prof. Schröter of Munich, and also water-vapor, as used in "vacuum ice machines." Several tables have been added to more completely compare the theoretical performance of the compression and absorption systems of ammonia machines.

The experimental determinations now available for the latent heat of ammonia

are, one by Regnault at 53.0 degrees Fahr., one by Strombeck at 64 degrees Fahr., and six by Denton at various temperatures between -10.7 and 87.7 . These agree with the theoretical forecast of Ledoux within from 1 to 5 per cent. Experiments on the latent heat of sulphur dioxide, between 14 and 68 degrees Fahr., by Mathias, Favre and Silbermann, and Chappuis, agree within 3 per cent. with Ledoux's calculations.

These results are detailed in a supplement to Ledoux's appendix.

Until the density of ammonia vapor at atmospheric pressure, upon which all theoretical formulæ for latent heat mainly depend, shall be better known, we may adhere to the values given in Ledoux's table of the properties of ammonia. The probable discrepancy of his figures is within the effect due the possible error of this density, and within the variation of duplicate experimental determinations of the latent heat. We have, therefore, not altered the values in

the tables for ammonia at the end of the work, notwithstanding that the value used by Ledoux for the constant C in Zeuner's fundamental formula (equation 86) is about 75 per cent. too large. We, however, give the values for ammonia, in our appendix, corrected for this error in the value of C , as obtained from tables in the latest edition of Zeuner's "Thermodynamik." In the appendix we have also compared the theoretical deductions of Wood and Peabody, and the experimental results of Regnault, Mathias, Favre and Silbermann.

It was the intention of the revisers to write such an introduction as might give to the practical reader, desirous of avoiding lengthy mathematical demonstrations, a comprehensive view of the theoretical and practical results arrived at in Ledoux's essay. Such an introduction having been nearly completed, proved to be much too bulky for the place intended for it, and it has therefore been decided to allow it more space, and to issue it

as a separate and subsequent volume. This will give full descriptions of the principal refrigerating plants and discuss the definite results of their use.

HOBOKEN, *July*, 1892.

4

A

I

	PAGE
Details of method of calculating cylinder super-heating.	xlvii
Table showing numerical results of calculation ...	liv
Indicator cards for air and ammonia	lv
Distribution of heat in absorption machine.....	lvi
Cold vs. dry compression systems.....	lviii

CHAPTER I.

General thermodynamic principles governing the action of refrigerating machinery.....	1-14
---	------

CHAPTER II.

Theoretical formulæ for performance of air machines with perfectly dry air and no clearance spaces....	15-26
Table of results of application of theoretical formulæ.....	27
Discussion of Table.....	28, 29
Theoretical formulæ for performance of air machines with no clearance spaces, but with air containing the average amount of moisture in the atmosphere.....	30-38
Table of results of application of formulæ..	39
Discussion of Table.....	40
Theoretical formulæ for performance of air machines with dry air and four per cent of clearance spaces.....	41-55
Table of results of application of formulæ..	56
Neutralization of loss of useful effect due to clearance spaces, by adjustment of degree of "cushioning".....	57-59
Effect of injecting an excess of water into compressing cylinder of air machine.....	60-69
Theoretical formulæ.....	60
Value of atomizing injection as in Colladon's compressors.....	65

	PAGE
Air experiments of Gause and Post.....	65
Table of results of application of formulæ...	69

CHAPTER III.

REFRIGERATING MACHINES EMPLOYING THE VAPOR OF VOLATILE LIQUIDS—COMPRESSION SYSTEMS.

Theoretical formulæ, for reversible or Carnot cycle, requiring use of an expansion cylinder.	70-79
Numerical examples for sulphur dioxide ...	79-84
" " " ammonia.....	85
" " " carbonic acid.....	86
" " " ether.....	88
Theoretical formulæ, for non-reversible cycle, using an expansion cock instead of an expan- sion cylinder, without superheating the gas....	89-98
Numerical examples for sulphur dioxide.....	94
" " " ammonia.....	94
" " " ether....	94
" " " carbonic acid.....	94
Theoretical formulæ, using expansion cock, and permitting the superheating of the gas during compression	95
General effect of superheating... ..	95-97
Establishment of constants in Zeuner's equa- tion for superheated sulphur dioxide and ammonia.....	98-104
Establishment of equation for adiabatic curve of compression of superheated am- monia and sulphur dioxide [Eq. 96]....	105-110
Formulæ for work represented by indicator card of compressor [Eq. 101].....	113
Formulæ for theoretical performance.....	115
Numerical examples and table of results of application of formulæ to sulphur di- oxide	118-122

	PAGE
Practical results obtained from sulphur dioxide machines.....	124
Numerical examples and theoretical table for ammonia	128-130
Practical results obtained from ammonia compression machines.....	131
Numerical examples for carbonic acid ...	131-134

CHAPTER IV.

AMMONIA ABSORPTION MACHINES.

Description of operation.....	135-139
General thermodynamic principles common to the compression and absorption types.....	139-146
Experimental tables of pressure and temperature from absorption machine generators....	146-149
Tables of theoretical performance of absorption machine neglecting heat abstracted at absorber, and assuming perfectly anhydrous gas circulated.....	149-152
Theoretical formulæ for performance of absorption machines including effect of water entrained with ammonia and loss of heat at absorber..	153-159
Comparative theoretical tables XII. to XVI., for absorption and compression machines.....	160-162

APPENDIX.

Establishment of theoretical formulæ for physical properties of ammonia and sulphur dioxide.	163-169
Ledoux's theoretical table for sulphur dioxide.....	172
“ “ “ “ ammonia.....	173
Schröter's “ “ “ carbonic acid.....	174
Zeuner's theoretical table for sulphur dioxide.	175
“ “ “ “ ammonia.	176
Résumé of experimental determinations of latent heat of refrigerating fluids.....	177-185

INTRODUCTION.

The units adopted to express the refrigerating effect throughout the text are, as follows:

1st. *Useful Effect*, which is used to represent the refrigeration in British thermal units, produced by a unit of substance at the lowest temperature of the cycle, divided by the foot-pounds of work required to compress this unit of substance without friction. The *useful effect* is therefore represented by the general expression

$$A \frac{Q_1}{Q - Q_1}.$$

Q_1 = heat abstracted from substance.

Q = heat given to substance, as from a bath of brine, or the refrigerating effect.

A = reciprocal of Joule's equivalent.

2d. The *Ice-melting Capacity* per pound of coal. This is used in all tables to represent the equivalent of the "useful effect," Q_1 being expressed in pounds of imaginary ice by dividing by 142.2 British thermal units, the latent heat of fusion of ice; and $Q - Q_1$ being expressed in pounds of coal, on the assumption that three pounds of coal is consumed per hour per indicated horsepower. This rate of coal consumption corresponds to the use of a good non-condensing automatic engine, having a boiler utilizing about 10,000 British thermal units per pound of coal. For more economical engines the figures in the tables are proportionately reduced.

3d. To represent the total refrigerating output of a machine the term "tons of ice-melting capacity" per 24 hours is used in the tables. This represents the equivalent of Q_1 multiplied by the number of units of the refrigerating substance circulated in 24 hours and divided by 2000 times the latent heat of fusion of ice.

The tables also give in gallons per "ton of ice-melting capacity" the amount of cooling water required to condense the refrigerating substance. When water must be bought at an expense of about one dollar per 1000 cubic feet, it is about equal to the coal as an element of cost, at a price of four dollars per ton.

It is to be understood that the performance of a machine, expressed in pounds or tons of "ice-melting capacity," does not mean that the refrigerating machine would make the same amount of actual ice, but rather that the cold produced is equivalent to the effect of the melting of ice at 32 degrees to water of the same temperature.

In order to measure the *ice-melting capacity*, as defined above, it is necessary that the refrigerating fluid, when at its lowest temperature, should be circulated in an insulated bath of non-freezable liquid, such as brine, and the latter, by circulation in contact with some source of heat, must vary in temperature through

a fixed range between the outlet and inlet of the brine reservoir. The quantity of brine circulated per unit of time, its range of temperature and its specific heat being then determined by measurement, the refrigerating effect per unit of coal required to operate the apparatus, or per 24 hours, may be stated as equivalent to the cold required to freeze water at 32 degrees Fahr. into ice at the same temperature. These results will then be directly comparable with the second and third units defined above.

In making artificial ice the water frozen is generally about 70 degrees in temperature when submitted to the refrigerating effect of a machine; second, the ice is chilled from 12 to 20 degrees below its freezing-point; third, there is a miscellaneous dissipation of cold, not exactly definable, from the exposure of the brine tank and the manipulation of the ice cans—therefore, the weight of actual ice made, multiplied by its latent heat of fusion, 142.2 thermal units, represents only about three-fourths of the cold pro-

duced in the brine by the refrigerating fluid * per indicated horse-power of the steam-engine driving the compressing pumps. Again, there is considerable fuel consumed, in ice-making, to operate the brine circulating pump, the condensing water and feed pumps, and to reboil, or purify, the condensed steam from which the ice is frozen. This fuel, together with that wasted in leakage and drip water, amounts to about one-half that required to drive the main steam-engine.† Hence, the pounds of actual ice manufactured from distilled water is just about half the equivalent of the refrigerating effect produced in the brine per indicated

* It is shown below that the cold produced in the brine may be only about three-fourths of that due to the latent heat of the weight of ammonia corresponding to the displacement of the compressors and the suction pressure density of the saturated vapor, as the heating of the ammonia by the walls of the cylinder makes the weight of ammonia usefully vaporized about 25% less than this theoretical amount, assuming no loss by clearance, leakage, or valve slip.

† When ice is made directly from natural water by means of the "plate system," about half of the fuel, used with distilled water, is saved by avoiding the re-boiling operation, and using steam expansively in a compound engine.

horse-power of the steam cylinders; that is, half the *ice-melting capacity* as defined in the second and third of the above units.

It is therefore to be understood that the term *ice melting capacity* means the cold produced in an insulated bath of brine, on the *assumption* that each 142.2 B. T. U. abstracted from the brine, represents one pound of ice.

COMPARISON OF THE RESULTS OF THEORY WITH EXPERIMENTAL MEASUREMENTS OF REFRIGERATING MACHINES IN PRACTICE.

Of the refrigerating fluids which have been, or which still are, used in practice, there are no records regarding air and ether which enable us to state the *ice-melting capacity*. Ether machines, used in India, are said to have produced about 6 pounds of actual ice per pound of fuel consumed, which would roughly prove that they realized as great a proportion of their theoretical efficiency as will be shown below to be obtainable from ammonia and

Class of machine

Ammonia Cold Co

Pictet Fl'd Dry Co

Bell-Coleman Air
Closed Cycle Air

Ammonia Dry Co

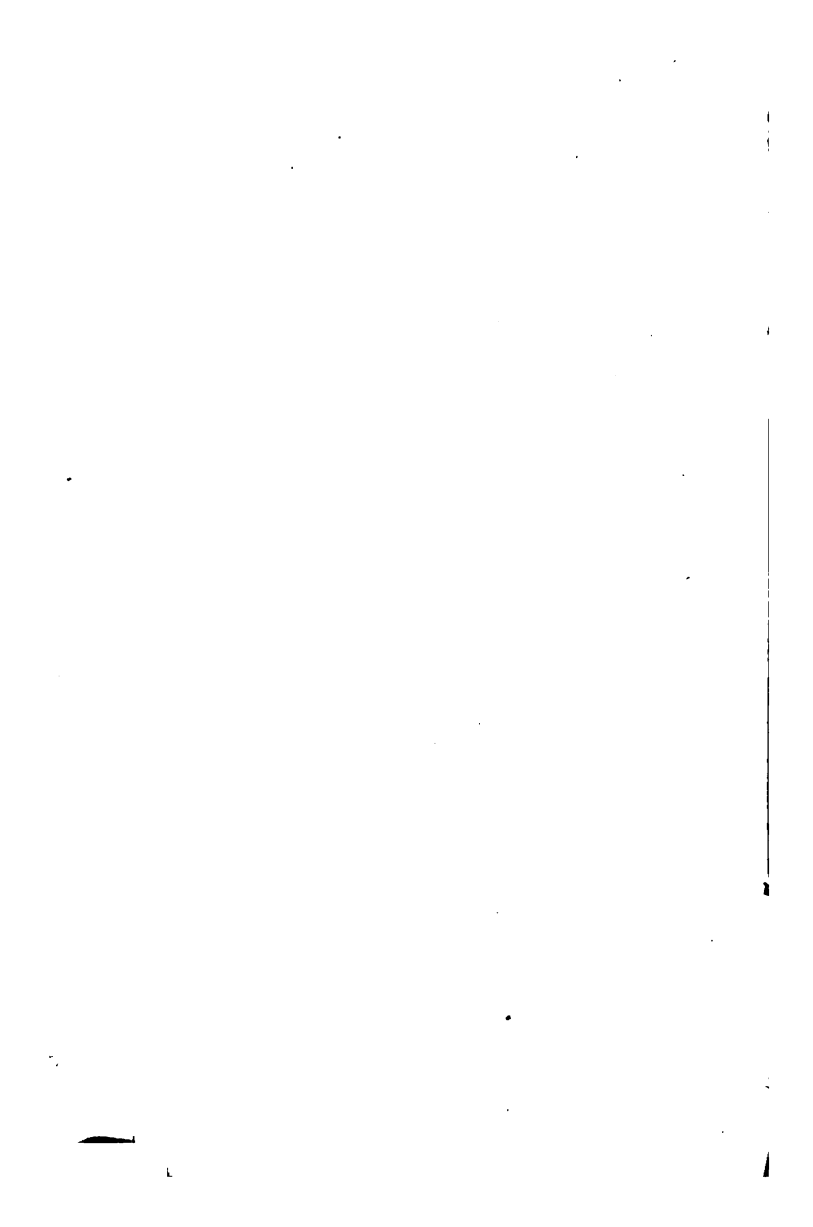
Ammonia Absor

* Temperature

Capacity in lb. of coal at 3 lb. per H. P. cylinder of machine. Evaporation of water per lb. of ice-melting machine.		no heat as-	Actual.	Difference between theoretical ice-melting capacity, no cylinder heating or friction, and actual ice-melting capacity. Per cent of theoretical with no friction.	Loss due to heating during aspira- tion of gas in the compres- sion cylinder and to radiation and superheating at brine tank. Per cent of theoretical amount with friction.	Actual, including resist-
17	18	19	20	21	22	23
23	40.63	30.8	19.1	54		
50	30.01	33.5	20.2	53		
44	22.03	37.1	25.2	50		
76	16.14	42.9	29.1	44		
68	19.07	36.0	28.5	77		
85	46.29	26.5	19.9	56		
56	33.23	31.3	21.9	56		
46	17.55	41.1	28.3	46		
70	33.77	33.1	22.9	50		
09	45.01	35.2	23.8	52		
48	33.07	39.9	22.2	24		
71	24.11	41.8	24.0	22		
36	17.47	42.2	25.2	20		
50	10.14	54.5	38.5	16		
88	16.05	36.2	23.1	31		
67	36.19	33.4	22.5	21		
01	26.24	34.6	25.0	21		
90	11.93	47.5	33.4	18		
17	33.04	39.5	22.6	23		
86	16.68	37.7	27.0	33		
30	9.86	54.2	39.5	17		
94	3.42	71.7	56.9	24		
8.1	3.0	80.	63.	81		
7.36	24.16	32.8	11.7	61		
9.78	14.52	37.4	22.7	57		
1.56	17.55	34.9	18.6	51		
6.94	23.31	30.5	13.5	71		
.....	20.1	47.8		

atic compression.

To f



sulphur dioxide. The ether machine is, however, obsolete, because the density of the vapor of ether, at the necessary working pressure, requires that the compressing cylinder shall be about 6 times larger than for sulphur dioxide, and 17 times larger than for ammonia. Air machines require about 1.2 times greater capacity of compressing cylinder, and are, as a whole, more cumbersome than ether machines, but they remain in use on ship-board, because the use of air incurs no such risk of destruction of provisions preserved by cold, as exists by danger from leakage when such chemicals as ammonia or sulphur dioxide are used.* There are also no records of the measurement of the performance of air machines by the use of a bath of brine, as described above, which enable the *ice-melting capacity* to be stated, the air, at its lowest temperature, being either discharged free into the chambers to be cooled, as in the

* Ammonia compression machines with brine circulation have, however, now been successfully introduced on some of the large Transatlantic liners.

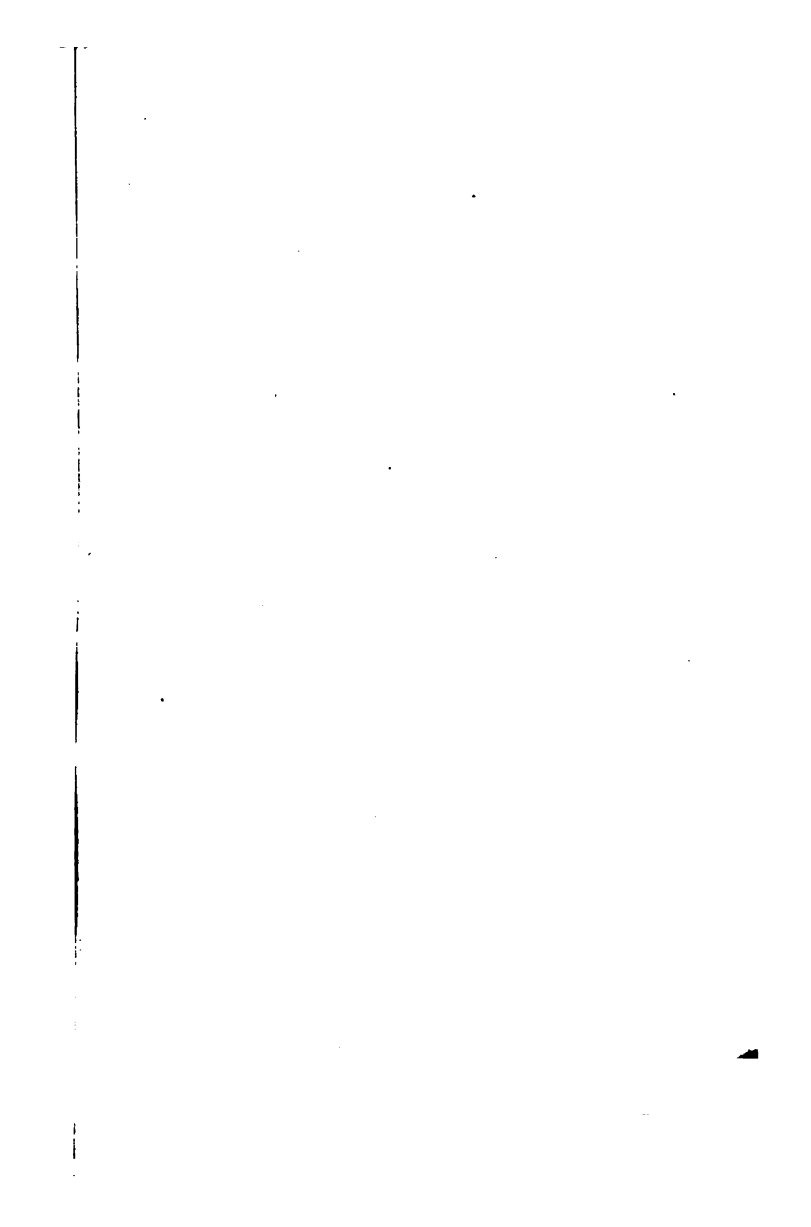
Bell-Coleman system for example, or circulated through closed pipes lodged within such chambers, as in the Allen Dense Air machine. All measurements of performance of air machines have, therefore, been based upon the determination of the range of temperature of the air between its entrance to the compressing cylinder and to the chamber to be cooled, combined with the weight of air circulated, the latter being computed from the displacement of the compression or the expansion cylinder, by the aid of indicator cards.

TESTS OF AIR MACHINES.

A good example of the above method of determination of performance of air machines is published by Prof. Schröter of Munich, for a Bell-Coleman machine, compressing air to about four atmospheres, and the result shows a refrigerating effect equivalent to 3.4 lbs. of *ice-melting capacity* per pound of coal required to drive the steam-engine, assuming 3 lbs. of coal per hour per horse-power

as the economy of the engine. This result is 43.1% of the theoretical result, computed upon thermodynamic principles, including an allowance for friction of mechanism of about 22% of the power of the steam cylinder, which was the actual friction as deduced from the indicator cards given by Prof. Schröter, samples of which are shown in Fig. A. A similar measurement of performance of a closed cycle air machine, compressing the air from 39 lbs. to 160 lbs. above the atmosphere, gives 3.0 lbs. of *ice-melting capacity*, or 37% of the theoretical results. Such a machine, by receiving the air at higher density than that corresponding to atmospheric pressure, has a proportionately smaller compressing cylinder than the ordinary type of air-machine. The details of both of these tests are given in lines 22 and 23 of Table I. The principal cause of the large difference between the theoretical and actual efficiency of air machines is the fact that the temperature of the air leaving the expansion cylinder is very much higher than

theory indicates for adiabatic expansion, even taking into account all possible influence of moisture. Thus, in the Bell-Coleman machine the theoretical temperature, due to the pressure to which the air was compressed, is -109 degrees Fahr., while the actual temperature was only -53 degrees. This result is not due to any deficiency in the adiabatic law of temperature, but probably to a rapid absorption of heat by the air by its contact with the metallic parts of the machine. Another cause of loss of theoretical effect is in the fall of pressure through wire-drawing of the air in passing from the compressing to the expanding cylinder. The amount of this is shown by the superposition of the compression and expansion indicator cards in Fig. A. It will be noticed that the compression line practically coincides with the adiabatic D C, notwithstanding that, by water injected into the compression cylinder the temperature of the air at its exit from this cylinder was only about 82 degrees Fahr.



RELATIVE PERFORMANCE OF AMMONIA COMPARED WITH

It is assumed in the calculation for both the ammonia and steam systems that the ammonia is equivalent to an evaporation of about 84 lbs. of water per pound above the atmosphere. The condensed steam from the generator is returned to the generator. The engine of the ammonia system is assumed to consume 26½ lbs. of water per hour per horsepower taken at 15 per cent of the work of compression without friction.

Number of comparison.	Condenser.		Refrigerat- ing coils.		Temp. of absorber, in degrees Fahr.	POUNDS OF ICE PER HOUR		Quantities of heat, in p. c. of amount furnished to the boiler, lost in exhaust of compression machine and friction.		
	Temperature, in degrees Fahr.	Absolute pressure, in lbs. per square inch.	Temperature, in degrees Fahr.	Absolute pressure, in lbs. per square inch.		Compress- ing ma- chine.	Using 3 lbs. of coal per hour per horse-power.	Using 1.6 lbs. of coal per hour per horse-power.	Absorption machine in coal per hour per H. P.	Using 1.6 lbs. of coal per hour per H. P.
1	2	3	4	5	6	7	8	9	10	
1	61.2	110.6	5	33.7	61.2	38.1	71.4	2.6	86.1	
2	59.0	106.0	5	33.7	59.0	39.8	74.6	2.6	86.1	
3	59.0	106.0	5	33.7	130.0	39.8	74.6	2.6	86.1	
4	59.0	106.0	22	16.9	59.0	23.4	43.9	2.6	86.1	
5	86.0	170.8	5	33.7	86.0	25.0	46.9	2.6	86.1	
6	86.0	170.8	5	33.7	130.0	25.0	46.9	2.6	86.1	
7	86.0	170.8	22	16.9	86.0	16.5	30.8	2.6	86.1	
8	86.0	170.8	22	16.9	130.0	16.5	30.8	2.6	86.1	
9	104.0	227.7	5	33.7	104.0	19.6	36.8	2.6	86.1	
10	104.0	227.7	22	16.9	104.0	13.5	25.3	2.6	86.1	

* Five per cent of water entrained in the ammonia figures given in the table.

AMMONIA MACHINES—ABSORPTION
TYPE.

Several tests of this type of machine have been published by Prof. Schröter of Munich, as a preliminary report of the Munich Polytechnic Commission; but the data do not permit the *ice-melting capacity* to be satisfactorily determined, as the machines were devoted to ice-making.

Allowing for the difference in the ice-melting and ice-producing capacities, the results obtained are, however, nearly the same as those given in Table I for a Pontefex absorption machine, in which the data is approximately complete for scientific purposes. (*Vide* Trans. Amer. Soc. Mech. Engrs., Vol. X., p. 792.) The ammonia was worked between 138 and 23 lbs. pressure above the atmosphere. The *ice-melting capacity* per pound of coal was 20.1 lbs. on a basis of boiler economy equivalent to 3 lbs. of steam per indicated horse-power in a good non-condensing steam-engine. This result realizes 52.2% of the theoretical

effect due to pure anhydrous ammonia, with no losses by imperfect action in any part of the apparatus. Detailed results are given in Table A and at the close of this Introduction. The principal losses are:

First. The steam to drive the ammonia circulating pump, which exhausted into the atmosphere, and used 16.4% of the total steam at the rate of about 150 lbs. per hour per horse-power of the pump.

Second. The heat carried off from the weak liquor by the water from the absorber, which was equivalent to 19.7% of the steam consumption.

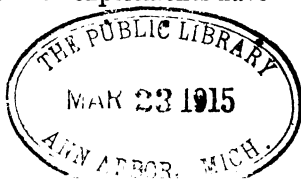
Third. Five per cent. of the weight of ammonia circulated is, on theoretical grounds,* believed to have been entrained with the ammonia throughout the cycle of the machine, which would cause a loss of 16.7% of the theoretical effect. The heat distribution is given in detail at the close of this Introduction.

* For argument on which this assumption is based, see Stevens' *Indicator*, January, 1892.

It is probable that more liberal proportions of the interchanger, or improvements in eliminating entrained water, and the saving of circulating-pump steam, possible by arranging this pump to exhaust into the generator, would enable the realization of about 65% of the theoretical effect, a figure which, as shown below, is attained by the compression type of machine under average conditions. For low back-pressures, theory indicates that the absorption machine is superior in economy to the compression machine, even when the latter has the advantage of being driven by a compound steam-engine. Table B illustrates this, but experiments are needed to test the truth of the conclusions. Such experiments are about to be made by the writers on a large Pontifex plant.

AMMONIA MACHINES—COMPRESSION TYPE.

Very complete determinations of *ice-melting capacity* of this type of machine are now available. Ten experiments have



been published by Prof. Schröter, as secretary of the Munich Commission, using a plant erected for experimental use. Lines 1 to 10 of Table I give the results in English units. The *ice-melting capacity* (col. 17) ranges from 46.29 to 16.14 lbs. of ice per lb. of coal, according as the suction pressure varies from about 45 to 8 lbs. above the atmosphere, this pressure being the condition which mainly controls the economy of compression machines. These results are equivalent to realizing from 72 to 57% (col. 19) of the theoretically perfect performance (col. 16). The higher per cents appear to occur with the higher suction pressures, indicating a greater loss from *cylinder heating*, (a phenomenon to be explained below [col. 20]), as the range of temperature of the gas in the compression cylinder is greater. These experiments were made with double-action compressors, operated on the "cold system" (see pages lviii and 95). Lines 24 to 27 show the results of experiments with single-acting compressors on one of the leading makes of

American compression machines operating on the "dry system." The percentage of theoretical effect realized ranges from 69.5 to 62.6%. The friction losses (col. 14) are higher for the American machine. The latter's higher efficiency may be attributed, therefore, to more perfect displacement. No clearance figures are published by Professor Schröter, but a comparison of the clearance action in the two types of machines, by means of the indicator cards, indicates that the less perfect displacement is not caused by excessive clearance spaces, but by the greater loss by *cylinder heating* (col. 20, Table I). This result is possibly ascribable to the smaller dimensions of the compression cylinders of the German machine, which affords greater surface in proportion to the volume of gas compressed per stroke, or, to a greater influence of the cylinder surfaces upon humid gas. The largest "ice-melting capacity" in the American machine is 24.16 lbs. This corresponds to the highest suction pressures used in

American practice, for such refrigeration as is required in beer storage cellars using the direct expansion system. The conditions most nearly corresponding to American brewery practice in the German tests are those in line 5, which give an "ice-melting capacity" of 19.07 lbs. Line 2, with a condensing pressure more favorable to economy of coal, but less favorable for economy of water, gives 30.01 lbs., but the per cent of perfect action realized is about 1% less than for the American machine, notwithstanding that the latter's loss by friction is 6% greater.

For the manufacture of artificial ice, the conditions of practice are those of lines 3 and 4, and lines 25 and 26. In the former, the condensing pressure used requires more expense for cooling water than is common in American practice. The *ice-melting capacity* is therefore greater in the German machine, being 22.03 and 16.14 lbs. against 17.55 and 14.52 for the American apparatus. The percentage of theoretical effect realized

again averages greatest in the single-acting machine, however, notwithstanding that the friction is less in the German apparatus.

CYLINDER HEATING.

In compression machines employing volatile vapors the principal cause of the loss of the theoretical result given in col. 16, Table I, is the heating of the ammonia, by the warm cylinder walls, during its entrance into the compressor, thereby expanding it, so that, to compress a pound of ammonia, a greater number of revolutions must be made by the compressing pumps than corresponds to the density of the ammonia gas as it issues from the brine tank (col. 16, Table I).

In the theoretical estimate of the refrigerating effect, it is assumed that the density, at exit from the brine tank, defines the displacement of compressor necessary to circulate a pound of gas through the refrigerating coils; but experimental measurement of the weight of ammonia circulated having

been made in connection with the tests of lines 24 to 27, Table I, by metering the liquid ammonia, it was found that from 15 to 30% less ammonia is circulated than is accounted for by multiplying the compressor displacement by the density corresponding to the pressure and temperature of the ammonia gas at its entrance to the compressor.

Hence, as the pressure does not vary, the density must have decreased by the rarefaction of the gas. It can be shown that the heat necessary to rarefy the gas can be lodged in a small thickness of the cylinder walls, and act in the same way as cylinder condensation in steam-engines. The theoretical mean effective pressure is practically the same for any range of pressure in the compression cylinder, whatever the density. Hence, the increase of power to operate the machine, for a given ton's capacity, is directly proportional to the increase of volume due to heating produced by the cylinder walls as the ammonia enters.

In the case of "cold compression" ma-

chines, or those in which liquid ammonia is injected in the cylinder in order to prevent superheating during compression, there is an increase of volume by cylinder heating, caused by the evaporation of a *portion* of the ammonia injected. In the case of dry compression the increase of volume is produced by superheating the gas.

Besides that due to heating at entrance to the cylinder the discrepancies between the theoretical and experimental results in ammonia compression machines are as follows:

1st. The liquid ammonia may be cooled by the condensing water below the boiling-point corresponding to condensing pressure. This increases the useful effect, because there will be a less per cent of each pound of ammonia vaporized in the brine tank to chill the substance from the temperature at which it arrives at the expansion cock to the boiling-point corresponding to back pressures. Such gain averages about 4% in the case of tests 24 to 27, Table I.

2d. The ammonia gas may be superheated before leaving the brine tank, above the boiling-point due to suction pressure. This makes a gain in the refrigerating effect obtained from a pound of the substance, amounting to about 2% in tests 24 to 27; but this is offset by the loss of effect due to the extra compression power required by the increase of volume caused by the superheating.

3d. The ammonia gas is superheated by absorbing heat from the atmosphere, in passing from the brine tank to the compressing cylinder. This in tests 24 to 27 causes a loss, by increase of compression power, of about 2%.

4th. There are also small losses by radiation from the brine tank, which are probably not more than 1%.

On the whole, therefore, the loss of effect in ammonia compression machines is practically entirely due to cylinder heating, which reduces the ice-melting capacity from 14 to 23% of the theoretical amount, this loss being roughly proportional to the range of

temperature in the compression cylinder. The exact method of calculating the loss due to "cylinder superheating" is given at the close of this Introduction.

TEMPERATURES OF COMPRESSED AMMONIA GAS.—INDICATOR CARDS.

A consequence of the theory of cylinder heating is that a large portion of the range of temperature due to compression is not shown by a thermometer located in the exit passage from the cylinder. The table on the next page illustrates this fact for the case of Dry Compression.

Thus in test 25, although the gas enters the cylinder at 25 degrees, in order to effect the rarefaction of the gas which, by the theory of cylinder superheating, is assumed to occur, heat stored in the cylinder walls acts to raise its temperature 132.6 degrees, as per line 2. Hence, when compression commences the gas is at 157.6 degrees (see columns 20 and 21, Table C). It then follows that the temperature at the end of compres-

**COMPARISON OF THEORETICAL AND ACTUAL
TEMPERATURE AT BEGINNING AND
END OF COMPRESSION.**

	Test 25.	Test 26.	Test 27.
	Degrees Fahren- heit.		
1. Temperature at entering cylinder, observed.....	25.0	10.1	34.0
2. Estimated superheating due cylinder walls.	132.6	116.9	76.8
3. Probable temp. at beginning of compression.....	157.6	127.0	110.8
4. At end of adiabatic compression	535.0	486.8	344.1
5. Temperature of gas leaving cylinder, observed.....	263.0	239.0	221.0
6. Loss of temperature during compression and ejection.....	272.0	197.8	123.1
7. Loss of temperature in excess of heat absorbed by cylinder, line 6 - line 2	139.4	80.9	46.3
8. Lbs. of ammonia circulated per minute as measured by meter	14.89	17.04	28.61
9. Heat removed by jackets per minute, B. T. U.	713	656	407
10. Temperature equivalent of line 9 = line 9 + (line 8 × 0.53).....	91.7	70.0	26.9
11. Loss of temperature unaccounted for, line 7 - line 10...	47.7	10.9	19.4

sion, if no heat was lost, would be

$$t_1 = (157.6 + 461) \left(\frac{P_1}{P_2} \right)^{0.24} - 461 = 535$$

degrees Fahr., as per line 4.

As the compression curve, Figs. B and C, agrees closely with the adiabatic law (eq. 96), it is probable that this tempera-

ture, line 4, is nearly attained during compression, but that during ejection the heat absorbed by the cylinder walls, and escaping through the jacket, reduces the temperature of the gas to that observed in line 5. Lines 7 to 11 show how nearly the heat abstracted by the water jackets surrounding the barrel of the compression cylinders accounts for the observed temperature of the gases at exit from the cylinder. The discrepancies in line 11 are fairly attributable to errors of measurement and losses by radiation, etc.

The indicator cards, Figs. B and C, show that a perfect action of the gas in the clearance spaces is obtainable, by sufficiently reducing the per cent of clearance, without the use of oil to fill these spaces. The perpendicularity of the fall of pressure in the clearance spaces is attributable to the cooling of the thin stratum of confined gas by the single-acting compressor piston, a special feature of which was the constant contact of the cold gas with the crank side of the piston.

**WEIGHT OF AMMONIA CALCULATED
FROM LATENT HEAT. TESTS 25, 26
AND 27.**

Using the values for latent heat of ammonia given in Ledoux's tables (page 173), the weight of ammonia circulated, calculated from the heat abstracted from the brine tank, in tests 25, 26, and 27, compares as follows with the weight as determined by meter :

	25.	26.	27.
Calculated lbs. per minute.....	14.34	17.64	28.61
Actual by meter "	14.89	17.04	28.31

It follows from these figures that Ledoux's latent heat values for low pressures are correct within five per cent, the limit of error of the experimental measurements. A similar agreement obtains for the condensing pressures (see table, page 185).

SULPHUR DIOXIDE OR PIOTET MACHINES.

No records are available for determination of the "ice-melting capacity" of machines using pure sulphur dioxide. This fluid is in use in American machines,

but in Europe it has given way to the "Pictet fluid," a mixture of about 97 per cent of sulphur dioxide and 3 per cent of carbonic acid. The presence of the carbonic acid affords a temperature about 14 Fahr. degrees lower than is obtained with pure sulphur dioxide at atmospheric pressure. The latent heat of this mixture has never been determined, but is assumed to be equal to that of pure sulphur dioxide. The Munich Commission have published tests of a "Pictet fluid" machine, erected, for testing, in their laboratory.

These tests are given in English units, in lines 11 to 21, Table I.

For brewery refrigerating conditions, line 17, we have 26.24 lbs. "ice-melting capacity," and for ice-making conditions, line 13, in which the "ice-melting capacity" is 17.47 lbs. These figures are practically as economical as those for ammonia, the per cent of theoretical effect realized ranging from 65.4 to 57.8. At extremely low temperatures—15° Fahr., lines 14 and 18—the per cent realized is

as low as 42.5. This is less than any ammonia experiment, but so low a temperature was not tried with this substance. The Pictet machine was at a small disadvantage in the German tests, in that the range of temperature of the circulating water was made the same in the two machines, and, in tests of the sulphur dioxide machine, less circulating water was used than in parallel tests of the ammonia machine, so that the condenser pressure was higher in the sulphur dioxide than it would have been had the machines been furnished with the same amount of condensing water. This disadvantage reduces the *ice-melting capacity per pound of coal* (col. 18), but does not affect the percentage of loss of theoretical effect (cols. 19 and 20).

On the whole, the tests do not indicate any serious inferiority of useful effect in the Pictet system. Theoretically there should be no practical difference between the fluids except greater bulk in the compression cylinder for the Pictet apparatus.

It will be observed that sulphur diox-

ide is also susceptible to loss of useful effect by *cylinder heating* (col. 20). A similar loss appears to occur in compressing air to several hundred pounds per square inch. But for pressures of about 100 lbs., the experiments of Messrs. Gause and Post at Hoboken show that the cylinder superheating, with jacket cooling, reduces the displacement only about 8%, when the air enters the compressing cylinder as directly as possible, through valves in the cylinder heads.

CARBONIC ACID MACHINES.

No experimental data of performance is available. To work between 64.4 and 5 degrees Fahr., the suction pressure must be 300, and the condensing pressure 800 lbs. per square inch.

The compression cylinder is four times less in volume than is necessary for ammonia.

The useful effect is considerably less than ammonia, if the liquid is worked through an expansion cock (see page 94).

WATER VAPOR MACHINES.

Such machines operate by water or brine being injected into a chamber maintained at a pressure of about one-tenth of a pound per square inch. In such a vacuum a portion of the water or brine becomes vapor, and the remainder is chilled in proportion to the latent heat of the amount vaporized. Ice made by this process is formed instantaneously, and is, therefore, white and opaque, although thoroughly solid and sweet. Attempts to utilize the vacuum principle for commercial purposes have produced two types of machines:

1st. The water vapor is removed by discharging it into a condenser like that of a steam-engine, from which it is removed as liquid, by an ordinary air pump, and discharged to waste. The water vapor is then compressed, as a superheated gas, from one-tenth to say one and one-half pounds per square inch, and the cycle is similar to that of the ammonia compression type.

2d. The water vapor is absorbed by sulphuric acid, which is pumped into a concentrator or distilling apparatus, where, under a vacuum, the water is driven from the acid in the form of vapor, and removed after condensation by the air pump maintaining the vacuum. The concentrated acid, at high temperature, runs back from the concentrator to the refrigerating chamber of the machine, giving up its heat to the cooler acid which is being delivered to the concentrator. This style of the apparatus is evidently very similar to the ammonia absorption machine, although not so simple.

One of the compression types of these machines has been built, having a double-acting compression cylinder, 20 feet in diameter, and a ten-foot stroke; but no results of performance are yet available.

The results of a test of a sulphuric acid machine of about ~~five~~ ^{eight} tons capacity have been recorded by Prof. Schröter in the report of the Polytechnic Commission of Munich. The useful effect was

seven

equivalent to about ~~three~~ pounds of ice-melting capacity per pound of coal, assuming each pound of the latter to utilize 10,000 B. T. U.

The machine worked under very unfavorable conditions, however, and the results are not representative of the possibilities of the system. Theoretically the economy should be not quite equal to that of ammonia for "ice-making conditions." The volume of vapor is about 150 times that of ammonia for ice-making conditions (see example, page 187).

ARTIFICIAL ICE MANUFACTURE.

Under summer conditions, with condensing water at 70 degrees Fahr., artificial ice machines use ammonia at about 190 lbs. above the atmosphere condenser pressure, and 15 lbs. suction pressure.

In a compression type of machine the useful circulation of ammonia, allowing for the effect of cylinder heating, is about 13 lbs. per hour per indicated horse-power of the steam cylinder. This weight of ammonia produces about 32 lbs. of ice, at

15 degrees, from water at 70 degrees Fahr. If the ice is made from distilled water, as in the "can system," the amount of the latter supplied by the boilers is about 33 per cent. greater than the weight of ice obtained. This excess represents steam escaping to the atmosphere, from the reboiler and steam condenser, to purify the distilled water, or free it from air; also, the loss through leaks and drips, and loss by melting of the ice in extricating it from the cans. The total steam consumed per horse-power is, therefore, about $32 \times 1.33 = 43.0$ lbs. About 7.0 lbs. of this covers the steam consumption of the steam-engines driving the centrifugal brine circulating pumps, the several cold-water pumps, and leakage, drips, etc. Consequently the main steam-engine must consume 36 lbs. of steam per hour per indicated steam horse-power, or else live steam must be condensed to supply the required amount of distilled water. There is, therefore, nothing to be gained by using steam at

high rates of expansion in the steam-engines, in making artificial ice from distilled water. If the cooling water for the ammonia coils and steam condenser is not too hard for use in the boilers, it may enter the latter at about 175 degrees Fahr., by restricting the quantity to $1\frac{1}{2}$ gallons per minute per ton of ice. With good coal $8\frac{1}{2}$ lbs. of feed-water may then be evaporated, on the average, per lb. of coal.

The ice made per pound of coal will then be $32 \div \frac{43.0}{8.5} = 6.0$ lbs. This corresponds with the results of average practice.

If ice is manufactured by the "plate system," no distilled water is used for freezing. Hence the water evaporated by the boilers may be reduced to the amount which will drive the steam motors, and the latter may use steam expansively to any extent consistent with the power required to compress the ammonia, operate the feed and filter pumps, and the hoist-

ing machinery. The latter may require about 15 per cent of the power needed for compressing the ammonia.

If a compound condensing steam-engine is used for driving the compressors, the steam per indicated steam horse-power, or per 32 lbs. of net ice, may be 14 lbs. per hour. The other motors at 50 lbs. of steam per horse-power will use 7.5 lbs. per hour, making the total consumption per steam horse-power of the compressor 21.5 lbs. Taking the evaporation at 8 lbs., the feed-water temperature being limited to about 110 degrees, the coal per horse-power is 2.7 lbs. per hour. The net ice per lb. of coal is then

about $\frac{32}{2.7} = 11.8$ lbs. The best results with "plate system" plants, using a compound steam-engine, have, thus far, afforded about $10\frac{1}{2}$ lbs. of ice per lb. of coal.

In the "plate system" the ice gradually forms, in from 9 to 14 days, to a thickness of about 14 inches, on hollow

plates 10×14 feet in area, in which the cooling fluid circulates.

In the "can system" the water is frozen in small blocks weighing about 300 lbs. each, and the process of freezing is completed in from 50 to 60 hours. The freezing-tank area occupied by the "plate system" is, therefore, about four times, and the cubic contents about twelve times, as much as is required in the "can system." The investment for the "plate" is about one-third greater than for the "can system." In the latter system ice is being drawn throughout the 24 hours, and the hoisting is done by hand-tackle. In the "plate system" the entire daily product is drawn, cut, and stored in a few hours, the hoisting being performed by power. The distribution of cost is as follows for the two systems, taking the cost for the "can" or distilled-water system as 100, which represents an actual cost of about one dollar and a quarter per net ton.

	Can system.	Plate system.
Hoisting and storing ice.....	14.2	2.8
Engineers, firemen, and coal-passer.....	15.0	13.9
Coal at \$3.50 per gross ton....	42.2	20.0
Water pumped directly from a natural source at 5 cts. per 1000 cubic feet.....	1.3	2.6
Interest and depreciation at 10 per cent....	24.6	32.7
Repairs.....	2.7	3.4
	<hr/> 100.00	<hr/> 75.4

A compound condensing engine is assumed to be used by the "plate system."

METHOD OF CALCULATING LOSS OF ECONOMY BY "CYLINDER SUPERHEATING."

The method of arriving at the per cents of loss by superheating (col. 20, Table I), is as follows :

In test No. 27, for example, suppose there were no losses by radiation, or superheating, or friction of mechanism—that is, suppose ammonia arrived at the expansion cock at exactly the boiling temperature for the condensing pressure, that it issued from the brine tank and passed to the inlet of the compressing cylinder at exactly the boiling tempera-

ture for the suction pressure; then, with no friction loss in mechanism, the ice-melting capacity per pound of coal, calculated as per examples, section 29, would be 33.54 lbs., as in col. 16, and including the actual loss of power by friction, given in col. 14, the ice-melting capacity per lb. of coal, is reduced to 23.31 lbs., as per col. 17. Now for these conditions the work of compressing the ammonia is represented by DBEG, Figs. B and C, and to circulate the 28.61 lbs. of ammonia per minute, shown by the meter, would require 49 revolutions per minute. As a matter of fact, however, 58.9 revolutions per minute were required to circulate this weight of ammonia. We therefore conclude that the gas is rarefied by superheating before compression, so that the volume of one pound is increased over that corresponding to the density for suction pressure, or 6.7 cubic feet, in the ratio of the above speeds—that is, as 1 to 1.202. Part of this superheating effect takes place in the brine tank, and

another part in the passage of the gas from the brine tank to the compressor. These are both known by measurement of temperatures, and their joint amount accounts for only a fraction of the increase of volume represented by the above ratio of revolutions. We therefore conceive that there is sufficient superheating influence by the cylinder walls to make up the deficiency, and compute its amount as follows :

Let T = temperature corresponding to suction pressure.

ΔT_1 = degrees superheating in brine tank.

ΔT_2 = degrees superheating between brine tank and compressor.

ΔT_3 = degrees superheating by cylinder walls.

By Zeuner's formulæ we have

$$V = \frac{BT}{p} - Cp^{n-1},$$

$$V_1 = \frac{B(T + \Delta T_1)}{p} - Cp^{n-1},$$

$$V_1 = \frac{B(T + \Delta T_1 + \Delta T_2)}{p} - Cp^{n-1},$$

$$V_2 = B(T + \Delta T_1 + \Delta T_2 + \Delta T_3) - Cp^{n-1}.$$

If the number of revolutions be N , then we have

$$\frac{V_1}{V} = \frac{N_1}{N}.$$

The total increase of work due to superheating in per cent of theoretical work, with no superheating, will be:

$$100 \frac{N_1 - N}{N} = 100 \frac{V_1 - V}{V}.$$

That due to superheating at brine tank will be:

$$100 \frac{N_1 - N}{N} =$$

$$100 \frac{V_1 - V}{V} = 100 \frac{V_2 - V}{V} \times \frac{V_1 - V}{V_2 - V} =$$

$$\left\{ \begin{array}{l} \text{Total increase of} \\ \text{work in per cent} \end{array} \right\} \times \frac{V_1 - V}{V_2 - V} =$$

$$\left\{ \begin{array}{l} \text{Total increase} \\ \text{of work in} \\ \text{per cent} \end{array} \right\} \times \frac{\Delta T_1}{\Delta T_1 + \Delta T_2 + \Delta T_3};$$

In a similar way we have for the losses at the other points:

Loss due to superheating between
brine tank and compressor in per
cent $\left. \vphantom{\begin{array}{l} \text{Total increase} \\ \text{of work in} \\ \text{per cent} \end{array}} \right\} =$

$$\left\{ \begin{array}{l} \text{Total increase} \\ \text{of work in} \\ \text{per cent} \end{array} \right\} \times \frac{\Delta T_2}{\Delta T_1 + \Delta T_2 + \Delta T_3};$$

Loss due to cylinder superheating =

$$\left\{ \begin{array}{l} \text{Total increase} \\ \text{of work in} \\ \text{per cent} \end{array} \right\} \times \frac{\Delta T_3}{\Delta T_1 + \Delta T_2 + \Delta T_3}.$$

In tests No. 26, 27, and 28 the total increase of work due to superheating can be calculated directly from the weight of anhydrous ammonia circulated. Then in test No. 27 we have

$$100 \frac{N_2 - N}{N} = 100 \frac{58.9 - 49}{49} = 0.202.$$

If the weight of anhydrous ammonia is not directly measured, we may determine its amount by means of the heat imparted to it at the brine tank. In order to do this the temperature of the anhydrous ammonia at entrance to the

brine tank must be measured, and also the temperature of the ammonia gas at exit from the tank. A convenient way of making this calculation is as follows:

Calculate the ice-melting capacity, taking into account the cooling of the anhydrous ammonia and superheating of gas at the brine tank, but assuming that the work remains the same as for the case of no superheating. The difference between this amount and the actual ice-melting capacity will be entirely due to the change of volume, or, if we call the calculated ice-melting capacity I_c and the actual I , then

$$\frac{V_s - V}{V} = \frac{I_c - I}{I}.$$

For test 27 we have

$$I_c = 28.05, \quad I = 23.31;$$

hence

$$\frac{V_s - V}{V} = .203,$$

which is about the same as is obtained by measuring the ammonia directly by a meter. In all the tests given in Table I, except 25 to 27, it is impossible to

obtain the increase of work by either of the above methods, because the weight of anhydrous ammonia is not known, which excludes the first method; and the temperature of the anhydrous ammonia entering the brine tank, and that of the gas leaving the same, is also not given, so that the second method cannot be applied. In Table I, therefore, the net loss is given, or the total loss due to increasing the work less the amount that is gained by cooling the liquid ammonia entering the brine tank below the temperature of the condenser and by superheating the gas at exit. This is given in per cent of the theoretical ice-melting effect with no superheating or cooling of the liquid ammonia at entrance to the brine tank below the temperature of the condenser.

The loss due to increase of power in per cent of I_o' will be:

$$100 \frac{I_c - I}{I_o'}.$$

But there is a gain of ice-melting capacity due to cooling the liquid am-

monia below the temperature of the condenser and to superheating at the brine tank, equal to $I_c - I'_c$; hence the net loss in per cent will be:

$$100 \frac{I_c - I}{I'_c} - 100 \frac{I_c - I'_c}{I'_c} = 100 \frac{I'_c - I}{I'_c}.$$

This is the quantity given in col. 17, Table I. For test No. 27 we have $I'_c = 26.94$; hence the total loss due to increase of power by superheating in per cent of I_c is:

$$100 \frac{28.05 - 23.31}{26.94} = 17.6\%,$$

which is the figure given in the table of the separate losses for tests 25, 26, and 27.

The net loss will be:

$$100 \frac{26.94 - 23.31}{26.94} = 13.5\%,$$

which is the figure given in col. 17, Table I.

Table C gives the detailed numerical steps upon which the calculation of the superheating effect depends.

REFRIGERATION MACHINES.

Temperature of gas entering compressor, in degrees Fahrenheit.		Temperature of gas at beginning of compression, calculated by substituting the actual volume of gas at beginning of compression deduced from Col. 19 in Zeuner's equation.		Superheating, in degrees Fahrenheit.		Losses due to superheating, in per cent of theoretical ice-melting capacity with no superheating. Obtained by calculating relative volumes by means of Zeuner's equation, and dividing Col. 18 in proportion to these volumes.
20	21	22	23	24		
35.0	157.7	24.3	11.4	132.7	At brine tank. Radiation between brine tank and compressor. Cylinder walls. Col. 21 - Col. 20.	
10.1	127.0	5.0	8.3	116.9		
34.0	110.8	14.3	5.2	76.8		
					At brine tank.	Radiation between brine tank and compressor. Cylinder walls.

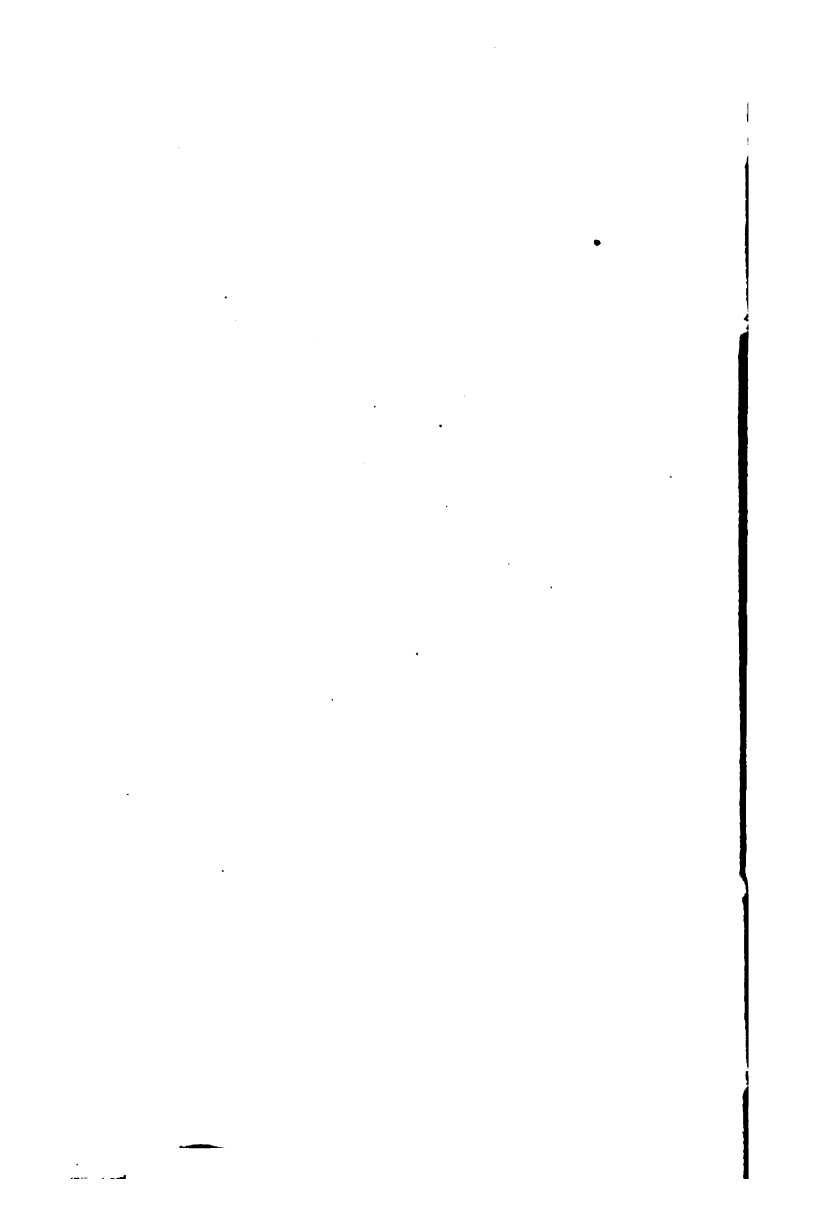


FIG. A.

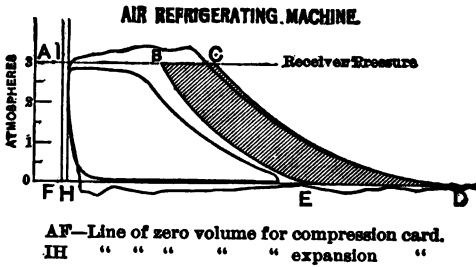
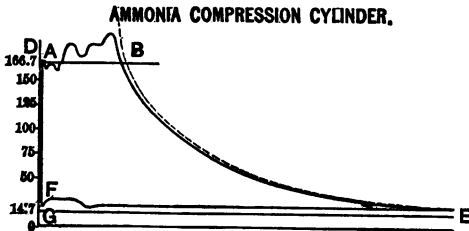


FIG. B.

Test No. 25.



Clearance $\frac{1}{10}$ per cent.

No oil injected.

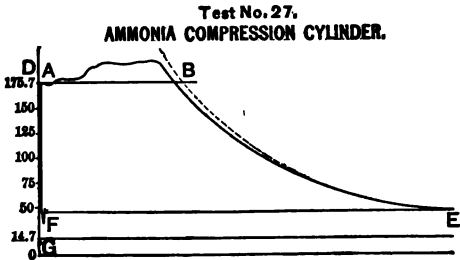
BE—Adiabatic curve—Eq. 96.

FIG. C.

Clearance A_1 per cent.

No oil injected.

BE—Adiabatic curve—Eq. 93.



COMPARISON OF THE THEORETICAL AND
EXPERIMENTAL ECONOMY IN AN
AMMONIA ABSORPTION MACHINE
(TEST NO. 27, TABLE I).

Ice-melting capacity per pound of
coal, obtained by test, assuming
that each pound of coal imparts
10,000 B. T. U. to the boiler..... 20.1 lbs.

Ice-melting capacity per pound of
coal, by Ledoux's equations, assum-
ing a perfect heater action, no drip
liquor, no water carried over by the
ammonia, and no losses by radia-
tion, and that the ammonia circu-

lating pump exhausts into the generator.*..... 88.5 lbs.

The calculated losses in per cent of the amount of heat imparted to the boiler, or $1,465,500 \times 1930 + 1614 = 1,752,400$ B. T. U., per hour are: †

Heater.....	19.7%
Drip liquor....	8.9%
5% water entrained with the ammonia.....	16.7%
Ammonia circulating pump.....	16.4%
Radiation ...	0.9%
	<hr/> 57.6%

There is a gain in refrigerating capacity in the machine tested, due to superheating the ammonia in the cooler, which is equivalent to..... 9.9%

Net loss.....	<hr/> 47.7%
---------------	-------------

* The conditions substituted in Ledoux's equations are: temperature of condenser, 80° Fahr.; temperature of cooler, 13° Fahr.; temperature of absorber, 130° Fahr.

† The per cents of loss, with the exception of that due to the 5% of water entrained with the ammonia, are found by dividing the heat expended, as shown in the text, by the total heat supplied to the boiler. The loss due to 5% of entrained water is found by comparing the efficiency of the machine when working with 5% of entrained water, with that obtained when no water is carried over with the ammonia.

Equivalent of losses in ice-melting capacity.....	$38.5 \times .477 = 18.4$ lbs.
Theoretical useful ice-melting capacity	$38.5 - 18.4 = 20.1$ lbs.

“COLD” vs. “DRY” SYSTEMS OF COMPRESSION.

In the “cold” system or “humid” system some of the ammonia entering the compression cylinder is liquid, so that the heat developed in the cylinder is absorbed by the liquid and the temperature of the ammonia thereby confined to the boiling-point due to the condenser pressure. No jacket is therefore required about the cylinder.

In the “dry” or “hot” system all ammonia entering the compressor is gaseous, and the temperature becomes by compression several hundred degrees greater than the boiling point due condenser pressure. A water jacket is therefore necessary to permit the cylinder to be properly lubricated. For comparative economy see discussion, page xxv.

ICE-MAKING MACHINES.

BY M. LEDOUX.

From Annales des Mines, 1878.

CHAPTER I.

§ 1. It has long been known that air is heated or cooled when compressed or dilated.

The mechanical theory of heat defines the conditions under which this heating or cooling is effected, and shows that these effects are proportioned to the external work performed by the air, with the restriction that in expanding the resistance overcome by the gas is always equal to the elastic force of the latter.

If t and t' represent successive temperatures of a unit weight of a permanent gas, which has been compressed or dilated under conditions above stated in

producing an amount of work (either resistant or motive) equal to W , we shall have

$$t - t' = \frac{A}{c} W$$

A being the reciprocal of the mechanical equivalent of heat $= 1/772.9$, and c being the specific heat of the gas at constant volume.

In a saturated vapor a part of the thermal equivalent of the external work is transformed into latent heat; the other part alone becomes sensible under the form of external heat.

This is expressed in the fundamental equation

$$c_1(t - t') + (x\rho - x'\rho') = AW,$$

in which c_1 is the specific heat of the liquid, x the proportion of vapor in the unit of weight of mixture of liquid and vapor, ρ the latent heat of the vapor, and W the external work accomplished.

We see from these equations that for the same quantity of heat transformed

into work, the range of temperatures must be greater with a gas than with saturated vapors.

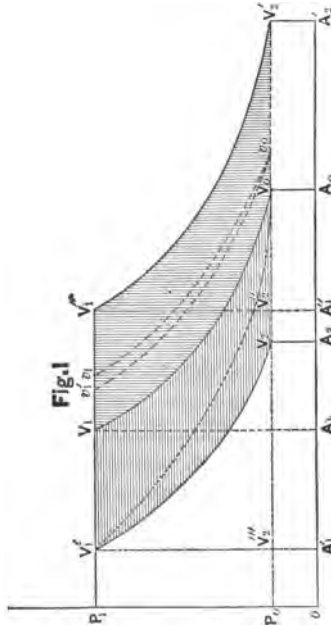
§ 2. Whether we employ a permanent gas or a vapor, the apparatus designed for the refrigerating effects is based upon the following series of operations:

Compress the gas or vapor by means of some external force, then relieve it of its heat so as to diminish its volume; next, cause this compressed gas or vapor to expand so as to produce mechanical work, and thus lower its temperature. The absorption of heat at this stage by the gas, in resuming its original condition, constitutes the refrigerating effect of the apparatus.

When the cooling takes place at constant pressure, the cycle of operations can be represented by the diagram Fig. 1, in which the abscissas represent volumes, and the ordinates pressures.

The gaseous body taken at the pressure P_0 and under the volume V_0 is compressed to the tension P_1 and the volume V_1 . It is then cooled under constant

pressure so that the volume V_1 becomes V_1' , then it is allowed to expand, the pressure P_1 becoming P_0 and the volume



changing from V_1' to V_2 . Finally it is brought to the original volume V_0 by

transferring heat to it under constant pressure. The area $V_0V_1V_1'V_2$ represents the work expended, and the line V_0V_2 the refrigerating effect obtained.

An inspection of the figure shows that a refrigerating machine is a heat engine reversed.

If instead of cooling the gas, to reduce it from the volume V_1 to V_1' , it be heated so as to assume the volume V_1'' greater than V_1 , an amount of work is obtained which is represented by the vertically shaded area $V_0V_2'V_1''V_1$; the heat expended is represented by the length V_1V_1'' .

It should be noticed that in the case of a permanent gas, the changes from volume V_1 to V_1' or V_1'' and from V_2 or V_2' to V_0 are accompanied by corresponding changes in temperature. In the case of a condensable vapor these changes are effected at a constant temperature, the addition or subtraction of heat taking effect in an evaporation of the liquid or a condensation of the vapor.

§ 3. From this similarity between heat

motors and freezing machines it results that all the equations deduced from the mechanical theory of heat to determine the performance of the first, apply equally to the second.

If Q_1 be the quantity of heat taken from or added to a given mass, of compressed gas or vapor, and Q the quantity of heat necessary to subtract from or add to the expanded mass in order to bring it to its initial state, T_0 and T_1 the absolute temperatures corresponding to the volumes V_0 and V_1 , and W the work, either active or resistant developed by the machine—the fundamental principle of the mechanical theory of heat, if the gas returns exactly to its primitive condition, affords the equation,

$$Q_1 - Q = AW.$$

If the cycle of changes is the so-called cycle of Carnot, that is to say, if the lines V_1V_0 , $V_1'V_2$, and $V_1''V_2'$ are adiabatic curves, then we have

$$\frac{Q}{T_0} = \frac{Q_1}{T_1} = \frac{Q_1 - Q}{T_1 - T_0}.$$

The quantity of work developed by a heat motor, under these circumstances, is for each heat unit, whatever the intermediate agent,

$$\frac{W}{Q_1} = \frac{1}{A} \cdot \frac{T_1 - T_0}{T_1}.$$

The efficiency depends upon the difference between the extremes of temperature.

The useful effect of a refrigerating machine depends upon the ratio between the heat units eliminated and the work expended in compressing and expanding.

It is measured by the ratio

$$\frac{Q}{W},$$

from which we have

$$\frac{Q}{W} = \frac{AQ}{Q_1 - Q} = A \frac{T_0}{T_1 - T_0} \dots (c)$$

This result is independent of the nature of the body employed.

Unlike the heat motors, the freezing machines possess the greatest efficiency

when the range of temperatures is small, and when the final temperature is elevated.

In a freezing machine employing a permanent gas the minimum temperature T_1 is different from the initial temperature T_0 , but we have

$$\frac{T_1}{T_0} = \frac{T_2}{T_1},$$

and combining this relation with eq. (c) we can write for the useful effect

$$\frac{Q}{W} = A \frac{T_2}{T_0 - T_1} \dots \dots (D)$$

Comparing (c) and (D) it is evident that the performance becomes less in proportion as we obtain lower final temperatures.

If the temperatures are the same, there is no theoretical advantage in employing a gas rather than a vapor in order to produce cold.

The choice of the intermediate body would be determined by practical considerations based on the physical character-

istics of the body, such as the greater or less facility for manipulating it; the extreme pressures required for the best effects, etc.

Air offers the double advantage that it is everywhere obtainable, and that we can vary at will the higher pressures, independent of the temperature of the refrigerant. But it is cumbersome,* and to produce a given useful effect the apparatus must be of large dimensions.

Liquefiable vapors, on the other hand, allow the use of smaller machines, but are obtained only at a greater or less cost.

Furthermore, the maximum pressure is determined beforehand by the temperature of the condenser, and depending on the nature of the volatile liquid; this pressure is often very high.

§ 4. The foregoing conclusions are based on the hypothesis that the compression and expansion follow the adiabatic lines V_0V_1 and $V_1'V_2$, that is to

* If worked in a closed cycle so that the pressure at T_0 is 4 atmospheres and the pressure at T_1 8 atmospheres, as in the "Allen Dense" Air Machine, the cumbrousness is reduced fourfold. See § 14, p. 40.

say, that the changes of volume and pressure follow the cycle of Carnot.

This hypothesis is realized when the cooling is accomplished outside of the compression cylinder and after the gas has been raised to the pressure P_1 .

If the compression be effected according to some cycle different from Carnot's, the efficiency, if it be a heat motor, would be diminished, but in a freezing machine it would be greater or less, depending upon the manner in which the successive operations were effected.

Suppose, for example, that instead of cooling the gaseous body outside the compression cylinder, it be done during compression within the cylinder in such a manner as to maintain a constant temperature. This hypothesis would be graphically represented in Fig. 1 by replacing the adiabatic curve V_0V_1 by the isothermal curve V_0V_1' . The work of resistance of the machine would then be represented by the curvilinear triangle $V_0V_1'V_2$, which is much smaller than the curvilinear rectangle $V_0V_1V_1'V_2$.

The quantity of negative heat produced, represented by the line V_0V_2 , remains the same. The efficiency of the freezing machine would be thus augmented as the resistant work of the motor would be less than the preceding case for the same quantity of negative heat produced.

§ 5. The useful effect is calculated in the following manner:

We suppose the compression to be made at a constant temperature, and that a unit of weight is considered. Then by Marriotte's Law we have $P_1V_1 = P_0V_0$.

The work of resistance to compression would be

$$W_r = P_0V_0 \cdot l \frac{V_0}{V_1} = RT_0 l \frac{V_0}{V_1}^*$$

and we shall have, as in the preceding case,

$$AW_r = Q_1.$$

R is a constant, the value of which is 53.35.

* The symbol l signifies the hyperbolic logarithm.

The gas dilating from the temperature T° to T_1 without gaining or losing heat, we shall have for the work of dilatation, inclusive of the work at full pressure during introduction,

$$AW_m = mkc(T_0 - T_1) = Q.$$

The useful effect is represented by

$$A \frac{Q}{Q_1 - Q}$$

and we have

$$\begin{aligned} \frac{Q'}{W_r - W_m} &= A \frac{Q}{Q_1 - Q} \\ &= \frac{kc(T_0 - T_1)}{RT_0 \left[\frac{P_1}{P_0} - \frac{kc}{A}(T_0 - T_1) \right]} \end{aligned}$$

We have also

$$\frac{c}{A} = \frac{R}{k-1}^*$$

* This gives $T_0(kc - c) = T_0RA = P_0V_0A$, which is the algebraic expression of the fact that the specific heat at constant pressure exceeds that for constant volume by the external work of expansion. See Rankine's Steam Engine, p. 318.

k is the ratio of specific heat at constant pressure to the specific heat at constant volume; this ratio is $= 1.41$ and is the same for all permanent gases.

It follows then that for isothermal compression

$$A \frac{Q}{Q_1 - Q} = A \frac{T_0 - T_1}{\left(\frac{k-1}{k}\right) T_0 \frac{P_1}{P_0} - (T_0 - T_1)}$$

If the compression follows an adiabatic curve, we shall have for the useful effect—calling T_1 the absolute final temperature of the compression—

$$A \frac{Q}{Q_1 - Q} = A \frac{T_0 - T_1}{T_1 - T_0 - (T_0 - T_1)}$$

and
$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{\frac{k-1}{k}}.$$

It is easy to show that

$$T_1 - T_0 \text{ or } T_0 \left\{ \left(\frac{P_1}{P_0}\right)^{\frac{k-1}{k}} - 1 \right\}$$

is greater than

$$\frac{k-1}{k} T_0 l \frac{P_1}{P_0},$$

and consequently that the useful effect in the second case is less than in the first.

The employment of air presents a certain theoretical advantage over volatile liquids, inasmuch as it admits of cooling to a certain extent during compression.*

We will now examine in succession some of the recently invented freezing machines (*machines à froid*). The Air Machine of M. Giffard; the Sulphurous Acid Machine of M. Pictet; the Ammonia Machine of M. Carré, and the compression type of Ammonia Machine.

* Such advantage is not practically available. See foot-note, Art. 19, p. 65.

CHAPTER II.

GIFFARD'S AIR MACHINE.

§ 7. This machine consists of a single-acting cylinder A, the piston of which is furnished with two valves opening inward. This cylinder is surrounded with a jacket, leaving a space within which circulates a current of cold water.

There is a second cylinder, B, also single-acting, and having a solid piston, with a diameter a little smaller than the first. At the bottom of this cylinder are two openings closed by valves, opening, one outward and the other inward, and operated by levers which are worked by cams on the driving shaft.

The pistons are driven by crank connections with the main shaft.

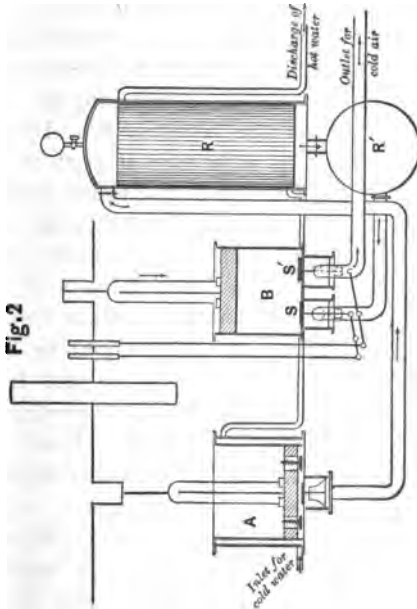
The condenser R is a surface condenser and receives a current of cold water from the envelope of the compressor cylinder A. A reservoir of wrought iron, R', is

connected with the condenser by a tube, and communicates also with the bottom of the expansion cylinder B.

§ 8. The air taken in at ordinary pressure is compressed in the cylinder A till it has the density of that in the reservoir; it is then allowed to flow into the condenser R and the reservoir R'. During this passage it loses a great part of the sensible heat which it attains during compression, and is brought nearly to the temperature of the surrounding air.

During this time the valve *s* of the cylinder B opens and permits a certain amount of air, equal in weight to that which is expelled from A, to pass from the reservoir into the cylinder, producing a certain amount of work. Then the valve *s* closes,—the air in the cylinder B expands, producing work again, which may be deducted from the work of compression and the temperature is lowered. When the piston B reaches the upper limit of its stroke, the valve *s'* opens and the cooled air, as the piston descends, escapes through the cold-air outlet.

The cooling experienced by the air, during compression, by contact with the



cooled sides of the cylinder is scarcely sensible.

The machine therefore acts under con-

ditions set forth in § 2, and we know that its useful effect cannot exceed the value

$$A \frac{T_0}{T_1 - T_0} \quad \text{or} \quad A \frac{T_2}{T_0 - T_2}.$$

By means of the adjustable cams we can regulate at will the action of the valves s and s' . If we shorten the time of admission into the cylinder B, the pressure will increase in the reservoir; for the amount flowing into B should be equal to that forced into the reservoir from A. The temperature of the air expelled will then be less. If, on the contrary, we increase the time of admission, the reservoir pressure will diminish, and the temperature of outflowing air will be increased.

The apparatus presents then this important peculiarity—that we can vary the useful effect of the machine at will, through wide limits.

As the air leaves B, at the pressure of the atmosphere, the minimum limit of pressure is established, below which the

expansion cannot be pushed, and which is controlled by the relative dimensions of the two cylinders.

We will proceed to calculate the cooling effect produced by this machine and the corresponding work required. We shall neglect at first the effect of waste spaces in the machine, and of watery vapor in the air.

§ 9. Let P_0 , t_0 and T_0 be the pressure and temperature (counted from absolute zero) of the air.

V_0 the volume described by the piston A.

V_1 the volume of air when at pressure P_1 .

V_1 is then the volume described by the piston during the outflow.

m = weight of air drawn into and ejected from the compressor.

P_1 , t_1 and T_1 the pressure and temperature of compressed air delivered from A.

V_1' , t_1' and T_1' the volume and temperature after passing into the condenser.

V , the total volume described by piston B.

P_1 , t_1 and T_1 the pressure and temperature of the air at the end of the course of this piston.

During compression the cooling by simple contact with the sides of the cylinder is insignificant. We shall neglect this and also assume that no heat is received from the sides of the cylinder B.

FIRST PERIOD: COMPRESSION.

§ 10. When air is compressed without losing or gaining heat, the pressure and temperature at each instant bear the relation to each other expressed by the equation

$$P_0 V_0^k = P_1 V_1^k, \quad (1)$$

in which k is the ratio of specific heat of constant pressure to the specific heat of constant volume.

$$k = \frac{0.23751}{0.16844} = 1.41.$$

Gay-Lussac's law affords,

$$P_0 V_0 = RmT_0 \quad (2)$$

and

$$P_1 V_1 = RmT_1. \quad (3)$$

From equations (1), (2), and (3), we deduce

$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{\frac{k-1}{k}} \quad (4)$$

$$\frac{T_1}{T_0} = \left(\frac{V_0}{V_1}\right)^{k-1}. \quad (5)$$

The work of the resistance to compression and outflow is*

$$W_r = \frac{k}{k-1}(P_1 V_1 - P_0 V_0). \quad (6)$$

We have elsewhere

$$\frac{k}{k-1} = \frac{kc}{AR}, \quad \dagger$$

* This is the value of the area $V_0 V_1 P_1 P_0 V_0$ or the equivalent of $P_1 V_1 + \int p dv - P_0 V_0$. The form of (6) should be committed to mind as it is constantly applied throughout this essay.

† See § 5, p. 16.

c being the specific heat of air of constant volume.

Equation (6) then becomes

$$W_r = \frac{mkc}{A}(T_1 - T_0). \quad (7)$$

SECOND PERIOD: COOLING.

The air is cooled in the condenser under constant pressure. The volume changes from V_1 to V_1' , and the temperature from t_1 to t_1' .

We have

$$V_1' = V_1 \frac{T_1'}{T_1}, \quad (8)$$

and the quantity of heat imparted to the water of the condenser is

$$Q_1 = mkc(T_1 - T_1'). \quad (9)$$

If $T_1' = T_0$ then $Q_1 = AW_r$.

THIRD PERIOD: EXPANSION.

The volume V_1' of air enters the cylinder B yielding an amount of work equal to $P_1 V_1'$. It expands from V_1' to V_2 .

without gain or loss of heat. We have then

$$P_1 V_1'^k = P_2 V_2^k, \quad (10)$$

$$P_1 V_1' = RmT_1', \quad (11)$$

$$P_2 V_2 = RmT_2; \quad (12)$$

whence

$$T_2 = T_1' \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}. \quad (13)$$

The work performed by the air is *

$$W_m = \frac{k}{k-1} (P_1 V_1' - P_2 V_2) \quad (14)$$

or

$$W_m = \frac{mkc}{A} (T_1' - T_2). \quad (15)$$

The resistances to be overcome by external force amount to

$$W_r - W_m = \frac{mkc}{A} [(T_1 - T_0) - (T_1' - T_2)]. \quad (16)$$

* If the machine works with maximum economy, the final pressure P_2 should be equal to P_0 , the atmospheric pressure.

The equations (10), (12), and (13) give

$$\text{or} \quad \left. \begin{aligned} \frac{V_2}{V_1'} &= \frac{V_0}{V_1} \\ \frac{V_2}{V_0} &= \frac{T_1'}{T_1} \end{aligned} \right\} \quad (17)$$

and

$$\frac{T_2}{T_1'} = \frac{T_0}{T_1}. \quad (18)$$

Equation (17) expresses the ratio which should exist between the volumes of the two cylinders, in order that the air be finally expelled at atmospheric pressure, after having been compressed by a force P_1 .

The negative heat Q , produced by the apparatus, is the quantity of heat necessary to restore the air from the temperature t_2 to the temperature t_0 , under constant pressure. Hence

$$\text{or} \quad \left. \begin{aligned} Q &= mkc(T_0 - T_2) \\ Q &= mkcT_0 \left(1 - \frac{T_1'}{T_1}\right) \end{aligned} \right\} \quad (19)$$

§ 11. Since a given weight of air is restored, at the end of the operation, to the same temperature and pressure it had at the beginning, it follows that it has been through a perfect cycle, and we have, from the mechanical theory of heat,

$$Q_1 = A(W_r - W_m) + Q.$$

The theoretical useful effect of the machine is, calling it u ,

$$u = \frac{Q}{W_r - W_m} = A \cdot \frac{T_0 - T_1}{(T_1 - T_1') - (T_0 - T_2)},$$

$$u = A \cdot \frac{1}{\frac{T_1 - T_1'}{T_0 - T_2} - 1},$$

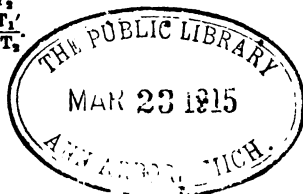
and as we have from equation (18)*

$$\frac{T_1 - T_1'}{T_0 - T_2} = \frac{T_1}{T_0} = \frac{T_1'}{T_2},$$

therefore,

$$u = A \cdot \frac{T_0}{T_1 - T_0} = A \cdot \frac{T_2}{T_1' - T_2}, \quad (20)$$

* Transform (18) to $\frac{T_1}{T_1'} = \frac{T_0}{T_2}$. Subtract unity from each side and solve for $\frac{T_1 - T_1'}{T_0 - T_2}$.



a result already found in § 3 by supposing $T_1' = T_0$. If $T_1' > T_0$, the useful effect is diminished.

The efficiency of the machine will increase as T_1 approaches in value to T_0 ; that is to say, as the air is urged at a lower pressure into the reservoir. But as we lower the pressure of working, the quantity of negative heat produced diminishes also and becomes nothing when $T_1' = T_1$.

The necessary driving power $W_r - W_m$, which we proceed to calculate, should be augmented by the passive resistances.

If we consider the refrigerating machine as composed of two distinct machines driven by the same shaft, we are led to consider that the work of the passive resistances is proportional not to the final work $W_r - W_m$ but rather to the sum of the work $W_r + W_m$ developed in the two cylinders. Considering the simplicity of the machine, the small amount of friction, and the absence of a stuffing box, we can admit that the work of the passive resistances should not ex-

REFRIGERATING POWER OF ONE CUBIC FOOT OF AIR AT 59°

m = weight of air.

kc = specific heat of air.

P_1	Temperature of air after adiabatic compression. t_1	Temperature of air as it leaves expansion cylinder. t_2	Range through which air is cooled. $t_0 - t_2$	Negative thermal units or heat abstracted from air. $mkc(t_0 - t_2)$	Work of compression cylinder without friction. W_r	Work of compression cylinder including friction. $1.08 W_r$	CONDENSING WATER.	
	Deg. F.	Deg. F.	Deg. F.	B.T.U.	Ft. lbs.	Ft. lbs.	Per ton of ice-melting capacity. Col. 19 + Col. 18.	Per minute per ton of ice-melting capacity in 24 hours. Col. 20 + 1440.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(20)	(21)
1½	123.9	+ 6.2	52.8	0.960	911.0	983.9	1270	0.88
2	174.8	- 31.2	90.2	1.639	1625.7	1755.7	1390	0.96
2½	217.3	- 58.1	117.1	2.128	2222.7	2400.6	1480	1.03
3	254.1	- 78.8	137.8	2.505	2739.9	2959.9	1570	1.09
3½	286.8	- 95.5	154.5	2.808	3199.5	3455.5	1640	1.14
4	316.4	- 109.4	168.4	3.059	3614.8	3903.8	1700	1.18
4½	343.4	- 121.2	180.2	3.275	3994.0	4313.0	1760	1.22

Columns 6 and 7 and 14 to 21 have been added.
To find tons melting capacity per 24 hours, calculate in column 18.
The ice-melting capacity per lb. of coal is found by dividing column 18 by the particular type of engine which drives the compressor.
The cut-off engine.
See face page 27.

ceed eight per cent of the above total work.

The resistance of the machine is then $1.08W_r - 0.92W_m$.*

Table I gives the amount of refrigeration obtained, and the work expended, by passing one cubic foot of dry air through the machine; the pressures in the reservoir varying from 1.5 to 4.5 atmospheres. The temperature of the external air is taken at 59° F. ; the temperature of the air leaving the condenser at 64.4° F. ; temperature of the water about 55.4° F. , $V_0 = 1 \text{ cu. ft.}$, $T_0 = 59 + 459.4 = 518.4$, and $m = 0.07653 \text{ lbs.}$

§ 12. An examination of the table shows the enormous influence that the passive resistances exert upon the efficiency of air machines. It is one of the consequences of the inherent cumbrousness which follows from the use of this body in a thermic machine.†

* This amount of friction waste is about three-fourths of that found in modern machines.

† For actual performance of air machines see Introduction.

The useful effect produced is not increased in proportion to the increase of pressure. It is of no advantage to employ pressures higher than about 4.5 atmospheres. Aside from the diminution of efficiency of the air at high pressures, a loss is occasioned by heat developed in the compressor, and which extends to other working parts of the machine. We have said above, that, with a given machine, we can vary at will the pressure P_1 by varying the length of time of the opening of the admission valve in the cylinder B. If the time be shortened, the pressure and the cooling effect are both increased; and if the time be increased, P_1 is diminished. It is necessary that we should vary at the same time the working of the emission valve, so that it opens at the moment when the piston shall have passed through a space equal to $V \frac{T_1'}{T_1}$ corresponding to the atmospheric pressure on the inside of the expansion cylinder.

A machine whose dimensions and ve-

locity are such that it uses 35,316 cubic feet of air per hour will produce an effect equivalent to freezing from 0.1194 to 0.4104 net tons of water, at 32° F. per hour, provided that the driving power varies from 3.95 to 33.5 horse-power.*

Practically, however, the useful effect of air machines is not so great as is indicated by the above table, as no account has yet been taken of watery vapor in the air, nor of lost spaces in the machine.

We will proceed to examine the influence of these two causes of loss.

INFLUENCE OF MOISTURE IN THE AIR.

§ 13. This influence is not to be neglected. The vapor contained in the air condenses on the sides of the expansion cylinder, and parts with its latent heat of vaporization so that the final temperature of the air is higher than it would have been if dry.

* The compressor would here be one of 18 in. diam. and 30 in. stroke running 65 revs. per minute, double acting.

Furthermore, the snow produced from this moisture accumulates around the orifice of the cold air outlet and we cannot readily utilize the cold which is required to produce it. For these two reasons, but especially for the latter, the moisture of the air causes a notable loss.

We proceed to calculate the volume and the temperature of the air at the end of the expansion under the supposition of a known hygrometric state of the atmosphere, from which we can easily deduce by the tables the pressure of the vapor p_0 and its weight μ_1 .

In the compression-cylinder the watery vapor not being near the saturation point, and exerting a feeble pressure, will behave nearly as a perfect gas; its volume and its temperature are represented by the relations $pv^k = \text{a constant}$, in which $k = 1.41$ and $pv = R'mT$;

$$R' = \frac{R}{0.622} = 85.7.$$

* The weight of one cubic foot of water vapor at atmospheric pressure is 0.622 of that of air.

The total pressure of air and vapor being represented by P , the pressure of the vapor being p , that of the air alone will be $P - p$, and we shall have, preserving our former notation,

$$P_1 V_1^k = P_0 V_0^k, \quad (21)$$

$$p_1 V_1^k = p_0 V_0^k, \quad (22)$$

$$(P_0 - p_0) V_0 = RmT_0, \quad (23)$$

$$p_0 V_0 = R' \mu_1 T_0, \quad (24)$$

$$(P_1 - p_1) V_1 = RmT_1, \quad (25)$$

$$p_1 V_1 = R' \mu_1 T_1. \quad (26)$$

The work of the resistance to compression is

$$\left. \begin{aligned} W_r &= \frac{k}{k-1} (P_1 V_1 - P_0 V_0) \\ \text{or} \\ W_r &= \frac{k}{A} (mc + \mu_1 c') (T_1 - T_0) \end{aligned} \right\} \quad (27)$$

c' is the specific heat, under constant volume, of the superheated vapor, its

value being 0.3407. After cooling, the volume becomes

$$V_1' = V_1 \frac{T_1'}{T_1}, \quad (28)$$

and we have

$$p_1 V_1' = R' \mu_1 T_1'.$$

From equations (21) and (22) we can deduce the pressure in the reservoir.

We can determine by examining a table of tensions of saturated steam whether the pressure p_1 is greater or less than the pressure which corresponds to the temperature T_1' . If it be less, the air will not be saturated with vapor when leaving the condenser, and the heat absorbed by the latter will be

$$Q_1 = k(mc + \mu_1 c')(T_1 - T_1').$$

If the pressure p_1 is greater than the pressure p_1' , corresponding to the temperature T_1' for saturated steam, there will be a condensation of some of the vapor in the condenser; the amount condensed will be

$$\mu_1(1 - x_1'),$$

and the pressure of the vapor entering into the cylinder B will be p_1' , that of the air being $P_1 - p_1'$.

We shall have also

$$x_1' = \frac{p_1'}{p_1} = \frac{p_1'}{p_0} \cdot \frac{P_0}{P_1}.$$

We see that the quantity of vapor not condensed by the cooling, and passing into the expansion cylinder, will continually diminish in proportion as the working pressure is raised. The influence of the humidity in the air will therefore be less as the pressure is made greater.

The weight of the mixture of air and vapor, which is $m + \mu_1$ if there is no condensation in the cooler, or $m + \mu_1 x_1'$ if there is a condensation, is carried into the cylinder B where it encounters the surfaces cooled during the preceding stroke. We can neglect the influence of these cold surfaces upon the air alone, but not upon the mixture of air and vapor. The latter is converted into frost

which releases a certain amount of heat to be imparted to the metal, and which during the expansion is restored to the air.

Supposing at first that there is no condensation in the cooler, there is conveyed to the cylinder a weight μ_1 of saturated, or nearly saturated, vapor at the temperature T_1' . We may assume, considering the very low temperature of the surfaces, that all the vapor is condensed here; it will disengage a quantity of heat C , which is approximately equal to $\mu_1(r_1' + 142.2)$. r_1' being the latent heat of the vapor corresponding to the temperature t_1' , and 142.2 the latent heat of water, which is released on freezing.

The heat C is gradually restored to the air during expansion.

The volume introduced into the cylinder is

$$V_1'' = \frac{RmT_1'}{P_1},$$

the pressure remaining P_1 .

The differential equation of the work is

$$\frac{c}{A}m dT + \frac{c_1}{A}\mu_1 dT - \frac{dC}{A} = -PdV$$

$$= -RmT \frac{dV}{V};$$

c_1 being the specific heat of ice, = 0.5, or

$$\left(\frac{c}{AR}m + \frac{c_1}{AR}\mu_1 \right) \frac{dT}{T} - \frac{dC}{ART} = -m \frac{dV}{V}.$$

We do not know the law of relation between C and T_1 , that is, the law according to which the heat, released from the water and ice formed, is communicated to the air. We are forced to make an hypothesis which is not rigorously exact, but which is a sufficiently near approximation.

We will suppose that the transmission is proportioned to the fall of temperature, and therefore that

$$dC = -\mu_1 \gamma dT,$$

in which

$$\gamma = \frac{r_1' + 142.2^*}{T_1' - T_2};$$

* γ is the loss of heat of each pound of vapor per degree of the entire range of temperature in the expansion cylinder.

whence we have

$$\frac{1}{AR}(mc + \mu_1 c_1 + \mu_1 \gamma) \frac{dT}{T} = -m \frac{dV}{V};$$

integrating we get

$$\frac{c}{AR} \left(1 + \frac{\mu_1 c_1 + \mu_1 \gamma}{mc} \right) l \frac{T_1'}{T_1} = l \frac{V_2}{V_1''}$$

or

$$\frac{1}{k-1} \left(1 + \frac{\mu_1 c_1 + \mu_1 \gamma}{mc} \right) l \frac{T_1'}{T_1} = l \frac{V_2}{V_1''}; \quad (29)$$

we have, furthermore,

$$P_0 V_1 = RmT_{1,},$$

$$P_1 V_1'' = RmT_1',$$

whence

$$\frac{T_1'}{T_1} = \frac{P_1 V_1''}{P_0 V_1}.$$

Equation (29) can then be written

$$\frac{1}{k-1} \left(k + \frac{\mu_1 c_1 + \mu_1 \gamma}{mc} \right) l \frac{T_1'}{T_1} = l \frac{P_1}{P_0}. \quad (30)$$

We can obtain the exact value of T_1 by successive approximations.

An approximate value for T_1 may be found by developing $\frac{T_1'}{T_2}$ into a series.

We have

$$\frac{T_1'}{T_2} = \frac{1 + \frac{T_1' - T_2}{T_1' + T_2}}{1 - \frac{T_1' - T_2}{T_1' + T_2}},$$

which may be developed into a series and all the terms after the first neglected, these being of the third power and over. Hence we have

$$\left[k + \frac{\mu_1 c_1}{mc} + \frac{\mu_1 (r' + 142.2)}{mc(T_1' - T_2)} \right] \frac{T_1' - T_2}{T_1' + T_2} = \frac{k - 1}{2} \ln \frac{P_1}{P_2}.$$

$$* \log \frac{1+x}{1-x} = \log (1+x) - \log (1-x)$$

$$\log (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}, \text{ etc.},$$

$$\log (1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}, \text{ etc.};$$

therefore $\log (1+x) - \log (1-x) = 2x$ if terms of the third power and over are disregarded.

Whence

$$T_2 = \frac{\left\{ mkc + \mu_1 c_1 - \frac{(k-1)mc}{2} \frac{P_1}{P_0} \right\} T_1' + \mu_1 (r_1' + 142.2)}{mkc + \mu_1 c_1 + \frac{(k-1)mc}{2} \frac{P_1}{P_0}}$$

Suppose now that condensation occurs in the cooler; we find by the tables the pressure of p_1' of saturated vapor of temperature T_1' , and we can deduce the weight of the vapor condensed in the cooler.

We shall have then

$$C = \mu_1 x_1' (r' + 142.2)$$

and

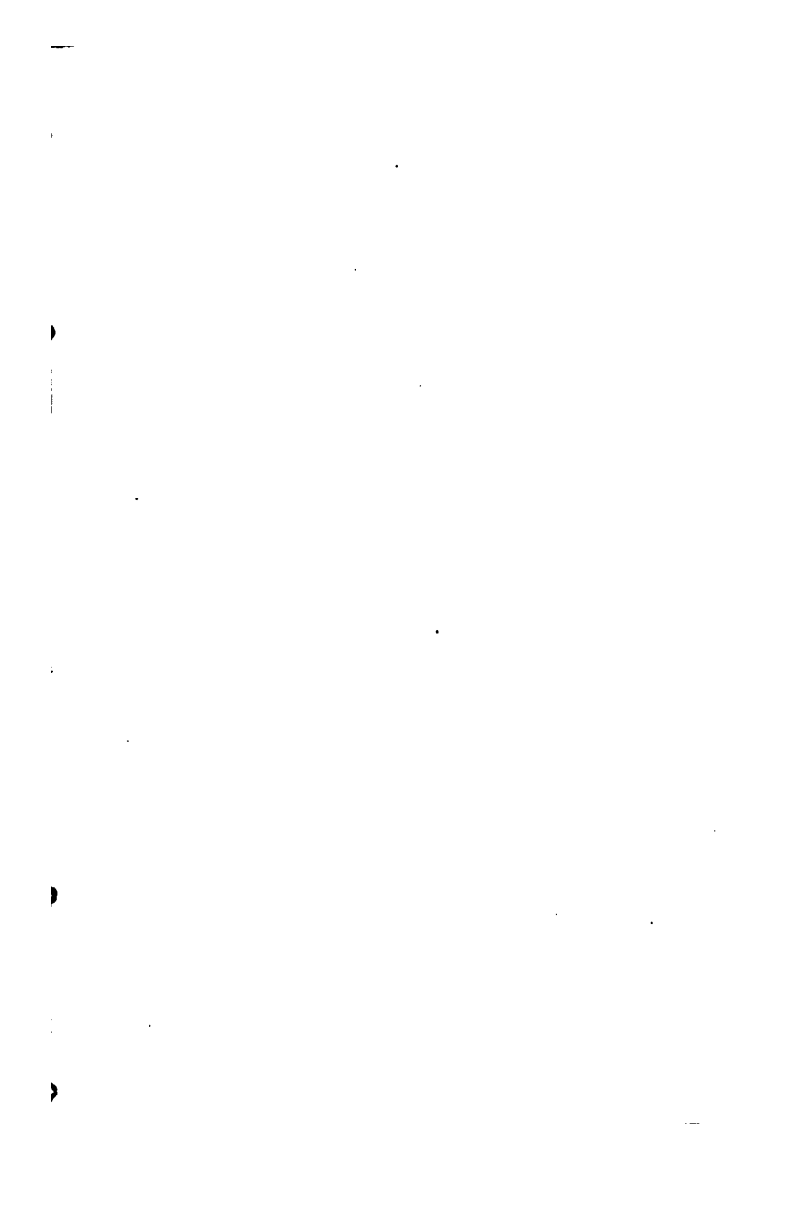
$$\gamma = x_1' \frac{r_1' + 142.2}{T_1' - T_2}.$$

The equations (29) and (30) apply in this case as in the preceding.

The quantity of disposable negative heat is

$$Q = mkc(T_0 - T_2), \quad (31)$$

since we suppose the negative heat of the snow formed to be lost.



3
9
4

**REFRIGERATING POWER OF ONE CUBIC INCH OF AIR AT 1 1/2 PRESSURE
CYLINDER FROM THE ATMOSPHERE AT 50 FACES
BEING ASSUMED ZERO. WEIGHT OF AIR 1.28.**

m = weight of air. kc = specific heat at constant pressure of air.

P ₁ Atmospheres.	Temperature of air after adiabatic compression.	Temperature of air as it leaves expansion cylinder.	Range through which air is cooled.	Negative thermal units or heat abstracted from air.	Work expended to operate machine without friction.	WATER.	
	t_1	t_2	$t_0 - t_2$	$mkc(t_0 - t_2)$	$Wr - Wm$	Col. 18 + Col. 16	Per minute per ton of ice-melting capacity in 24 hours. Col. 19 + 1440.
	Deg.F.	Deg.F.	Deg.F.	B.T.U.	Ft.-lbs.		Gals.
(1)	(2)	(3)	(4)	(5)	(6)		(20)
2	174.8	— 7.1	66.1	1.191	257.4	0	1.33
2½	217.3	— 35.0	94.0	1.696	468.2	0	1.30
3	254.1	— 59.4	118.4	2.134	713.9	0	1.30
3½	286.8	— 78.8	137.8	2.483	940.4	0	1.33
4	316.4	— 95.0	154.0	2.775	1163.9	0	1.35
4½	343.4	— 108.9	167.9	3.026	1873.6	0	1.37

Columns 12 to 16 and 18 to 20 have been added.
Values for $P_1 = 1 \frac{1}{2}$ atmospheres are given in a cylinder at this pressure will not be frozen until expansion immediately on entering the cylinder.

* Assuming 3 l.

† Q_1 is approximately page 39.

Finally the work produced by the expansion is

$$W_m = P_1 V_1'' + \frac{mc}{A} \left(1 + \frac{\mu_1 c_1 + \mu_1 \gamma}{mc} \right) (T_1' - T_2) - P_0 V_2 \quad (32)$$

or

$$W_m = \frac{mkc + \mu_1(c_1 + \gamma)}{A} (T_1' - T_2). \quad (33)$$

If there is a condensation in the cooler, we should replace μ_1 in equations (32) and (33) by $\mu_1 x_1'$.

§ 14. Table II gives the cooling and detailed effects obtained from one cubic foot of air supposing it half saturated and at a temperature of 59° F. The weight of the air is then .07589 instead of .07653 lbs., which is the weight of dry air at this temperature.

We have also* $p_0 = .123$ lbs. per sq. in. and $\mu_1 = .000391$ lbs.

* The tension of water vapor at 59° F. is 0.246 lbs. per sq. in. This corresponds to a hygrometric state of unity, or complete saturation. Hence 0.123 lbs. corresponds to half saturation.

The value of μ_1 is the weight of one half a cu. ft. of steam at 59° F. See table, p. 386, D. K. Clark's Manual.

In comparing this table with Table I, we see that the influence of the humidity of the air upon the results obtained is the greater when the pressure is low. We have made a similar remark in reference to the passive resistances. The theoretical advantage, therefore, of low pressures is practically much diminished by these causes of loss.

It is possible to neutralize almost completely the influence of moisture in the air. To accomplish this it would suffice to employ the air after it had produced its cooling effect and had parted with its moisture. It would be necessary to make the refrigerating machine a closed * machine, making the same quantity of air serve indefinitely. The cooling would be produced by causing the cooled air to pass through an apparatus surrounded by some liquid not easily frozen, such as a solution of calcium or magnesium chloride. A part of the negative calories

* Such a cycle is accomplished in the Allen Dense Air Machine, in which the influence of moisture is eliminated as suggested above. See foot-note, § 3, p. 9.

would thus be used, as well as by direct contact, and so many as are not used would not be lost, as the air passes directly to the compressor A, not at 59° F. as before, but at 17.6° or 14° F. of temperature. We think that it is only in this way that we can improve the air machine so that it can compare favorably with the machines using a liquefiable gas.

INFLUENCE OF WASTE SPACES.

§ 15. We will suppose the air to be dry in order to avoid complexity in our calculations.

Preserving our previous notation and calling v the amount of useless space in the compression cylinder, v' that of the expansion cylinder, and μ the weight of air enclosed in the space v at the end of the compression, we have:

$$P_0(V_0 + v)^k = P_1(V_1 + v)^k, \quad (34)$$

$$P_0(V_0 + v) = R(m + \mu)T_0, \quad (35)$$

$$P_1V_1 = RmT_1, \quad (36)$$

$$P_1v = R\mu T_1, \quad (37)$$

m being the weight of dry air driven out of the compressor.

Equations (34), (35), (36), and (37) give by elimination of μ

$$T_1 = T_0 \left(\frac{V_0 + v}{V_1 + v} \right)^{k-1}, \quad (38)$$

and

$$T_1 = T_0 \left(\frac{P_1}{P_0} \right)^{\frac{k-1}{k}} \quad (39)$$

The work of resistance to compression, taking account of the work restored to the piston by the air in the waste space expanding from P_1 to P_0 , is,

$$W_r = \frac{k}{k-1} (P_1 V_1 - P_0 V_0) * \\ + \frac{k}{k-1} P_0 v \frac{V_0 - V_1}{V_1 + v}. \quad (40)$$

* Referring to Fig. 1, p. 4, let OA_1' represent the clearance volume v . Then

$$Wr = V_0 V_1 V_1' V_2 = V_0 V_1 P_1 P_0 - V_2 V_1' P_1 P_0.$$

By the general form, equation (6), § 10, we have, therefore,

$$Wr = \frac{k}{k-1} [P_1(V_1 + v) - P_0(V_0 + v)] \\ - \frac{k}{k-1} \left[P_1 v - P_0 \left(\frac{P_1}{P_0} \right)^{\frac{1}{k}} v \right] \\ = \frac{k}{k-1} (P_1 V_1 - P_0 V_0) + \frac{k}{k-1} P_0 v \left(\frac{V_0 + v}{V_1 + v} - 1 \right).$$

For the cooling period,

$$V_1' = V_1 \frac{T_1'}{T_1}, \quad (41)$$

and

$$P_1 V_1' = RmT_1'. \quad (42)$$

The heat Q_1 absorbed by the water of the condenser is,

$$Q_1 = mkc(T_1 - T_1'). \quad (43)$$

PERIOD OF EXPANSION.—The air coming from the reservoir R' under pressure P_1 and at the temperature T_1' , should at the moment of opening of the inlet valve cause the air in the waste space, whose volume is v' , to change its pressure from P_0 to P_1 . This influences the temperature T_1'' of the contents of the clearance spaces and also the weight m' of the air which passes from the reservoir into the waste space.

The dimensions of the reservoir being very large in comparison to the waste spaces, we may assume that no change occurs either in temperature or pressure of the reservoir, while the waste spaces are filled with air at the pressure P_1 .

Call μ' the weight of the air enclosed in the waste space at the moment that the inlet valve opens. We then have

$$P_1 v' = R \mu' T_2; \quad (44)$$

T_2 being the final temperature of the expanded air.

The stored up work of this air is: *

$$\frac{c}{A} \mu' T_2.$$

The weight m' of air entering the waste space having a temperature T_1' and a pressure P_1 has a stored energy of

$$\frac{c}{A} m' T_1'.$$

After the waste space is filled, the stored up energy of the total quantity of air $m' + \mu'$ contained there is

$$\frac{c}{A} (m' + \mu') T_1'',$$

and we have, furthermore,

$$P_1 v' = R (m' + \mu') T_1''. \quad (45)$$

* This "stored-up work" is equivalent to the "intrinsic energy." Rankine Steam Engine, Art. 247.

As we suppose there is neither loss nor gain of heat from the exterior, the difference between the stored energy of the mixture after the mass m' is introduced, and the sum of the stored energies of the masses m' and μ' before mixing is equal to the external work performed.

Let v_1' equal the volume of m' before its introduction into the cylinder under pressure P_1 and temperature T_1' . Then the exterior work is

$$P_1 v_1' = Rm'T_1'.$$

We have also*

$$-\frac{c}{A}\mu'T_2 - \frac{c}{A}m'T_1' + \frac{c}{A}(m' + \mu')T_1'' = Rm'T_1'. \quad (45a)$$

* The equation [45a] is possibly more clearly derived as follows:—By the assumption, page 43, viz.: that the reservoir suffers no change of pressure or temperature during the exit from it of the volume v_1' necessary to fill the clearance space, we may say that the clearance space of the expansion cylinder contains the following amounts of energy expressed in mechanical units:

$\frac{c}{A}\mu'T_2$ —Remaining in the space after exhaust stroke
of expansion piston.

$\frac{c}{A}m'T_1'$ —Contained in the volume v_1' as it leaves the
reservoir.

Replacing $\frac{c}{A}$ by $\frac{R}{k-1}$ * and combining with equations (44) and (45)

$$m' = \frac{(P_1 - P_0)v'}{kRT_1'}, \dagger \quad (46)$$

$P_1v_1' - \left\{ \begin{array}{l} \text{Work done upon the air } v_1' \text{ in ejecting it} \\ \text{from the receiver. This work is part of} \\ \text{that exerted by the compressor piston dur-} \\ \text{ing the ejection of air from the compressor.} \end{array} \right.$

Since the piston of the expansion cylinder is stationary during this action, P_1v_1' appears in superheating effect.

The total energy is the sum of these items which must equal the expression $\frac{c}{A}(m' + \mu')T_1''$.

Therefore

$$P_1v_1' + \frac{c}{A}\mu'T_2 + \frac{c}{A}m'T_1' = \frac{c}{A}(m' + \mu')T_1'';$$

whence we have (45a).

* See foot-note, § 5, page 12.

† Substituting $\frac{R}{k-1}$ for $\frac{c}{A}$ in (45a) we have the relation

$$\frac{P_1 - P_0}{k-1}v' = Rm'T_1' \left(1 + \frac{1}{k-1}\right) = Rm'T_1' \frac{k}{k-1},$$

whence

$$m' = \frac{P_1 - P_0}{kRT_1'}v'.$$

$$\left. \begin{aligned} \text{and } T_1'' &= \frac{km'T_1' + \mu'T_2}{m' + \mu'}, * \\ \text{or } T_1'' &= \frac{kP_1T_1'T_2}{P_1T_2 + P_0(kT_1' - T_2)} \end{aligned} \right\} (47)$$

When the inlet valve closes, the piston has described a volume V_1'' , which has been filled by the weight m'' of air at pressure P_1 and temperature T_1' . We have then,

$$m' + m'' = m. \quad (47a)$$

There is no external work performed upon the total mass of air, since the negative work of the piston P_1V_1'' is exactly equal to the positive work exerted by the air of the reservoir. The

* In (45a) substitute $(k-1)\frac{C}{A}$ for R .

Solve for T_1'' and we have

$$T_1'' = \frac{km'T_1' + \mu'T_2}{m' + \mu'}.$$

In this equation put for m' its value in (46) and for μ' its value from (44) and we have

$$T_1'' = \frac{\frac{P_1 - P_0v'}{R} + \frac{P_0v'}{R}}{\frac{P_1 - P_0v'}{kRT_1'} + \frac{P_0v'}{RT_2}} = \frac{P_1kT_1'T_2}{P_1T_2 + P_0(kT_1' - T_2)}.$$

weights and temperatures of the air at the beginning and the end of the introduction possess the following relations:

$$\begin{aligned}\frac{c}{A}(m' + \mu)T_1'' + \frac{c}{A}m''T_1' \\ = \frac{c}{A}(m' + m'' + \mu')T_1'''.\end{aligned}$$

T_1''' being the temperature at the end of the introduction.

This equation gives,

$$T_1''' = \frac{(m' + \mu')T_1'' + m''T_1'}{m + \mu'}, \quad (47b)$$

* From (45)

$$(m' + \mu')T_1'' = \frac{P_1 v'}{R}.$$

From (46) and (47a)

$$m'' = m - m' = m - \frac{P_1 - P_0}{kRT_1'} v'$$

Substituting these in (47b) and reducing we have

$$T_1''' = \frac{P_1(V_1' + v') - \frac{1}{k}(P_1 - P_0)v'}{R(m + \mu')}.$$

By (42), $m = \frac{P_1 V_1'}{RT_1'}.$

By (44), $\mu' = \frac{P_0 v'}{RT_2},$

hence $m + \mu' = \frac{P_1 V_1' T_2 + P_0 v' T_1'}{RT_1' T_2},$

and this substituted in the above value of T_1''' gives (48).

or

$$T_1''' = \frac{P_1(V_1' + v') - \frac{1}{k}(P_1 - P_0)v'}{P_1 V_1' T_1 + P_0 v' T_1'} T_1' T_2; \quad (48)$$

we also have

$$P_1(V_1'' + v') = R(m + \mu')T_1''',$$

or substituting T_1''' from (48),

$$P_1(V_1'' + v') = P_1(V_1' + v') - \frac{1}{k}(P_1 - P_0)v'; * \quad (49)$$

* (49) gives

$$V_1'' + v' = V_1' + v' - \frac{P_1 - P_0}{kP_1}v' \quad (49a)$$

But from (46),

$$\frac{P_1 - P_0}{k}v' = Rm'T_1' = P_1v_1',$$

whence

$$v_1' = \frac{P_1 - P_0}{kP_1}v'$$

which is the volume entering the clearance space v' from the receiver at the pressure P_1 . Consequently

$$v' - \frac{P_1 - P_0}{kP_1}v'$$

is the volume of the air remaining in the clearance space of expansion cylinder *after* it is compressed to P_1 . Hence (49a) expresses the fact that the volume

V_1' is given by equations (38) and (41). Equations (49) gives the value of V_1'' .

The inlet valve being closed, the mass of air $m + \mu$ which is at pressure P_1 and temperature T_1''' expands without gain or loss of heat since we neglect the influence of the sides of cylinder. At the end of the stroke, this volume becomes $V_2 + v'$, its temperature T_2 and its pressure P_2 . We have then

$$\left. \begin{aligned} P_2(V_2 + v')^k &= P_1(V_1'' + v')^k, \\ \text{or} \\ P_2(V_2 + v')^k &= P_1 \\ &\left\{ (V_1' + v') - \frac{1}{k} \left(1 - \frac{P_0}{P_1} \right) v' \right\}^k \end{aligned} \right\} \quad (50)$$

and

$$P_2(V_2 + v') = R(m + \mu')T_2. \quad (51)$$

Equations (50) and (51) give V_2 and T_2 if P_2 is known, or P_2 and T_2 if V_2 is known; this latter being the volume described by the piston of cylinder B.

present at cut-off in the expansion cylinder is the sum of the volume V_1' ejected from the receiver and the volume left in the clearance spaces *after* this volume is compressed to P_1 . Eq. (49) may be written directly from (46) by aid of this conception.

We have

$$T_2 = T_1''' \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}. \quad (51a)$$

When there is no waste space we have

$$T_2 = T_1' \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}. \quad (51b)$$

As T_1''' is greater* than T_1' , it results that for a given weight of air passed through the machine, at a given working pressure, the final temperature of the expanded air would be higher, and consequently the number of negative heat units produced would be less than if there had been no waste spaces.

* The truth of this statement appears by substitution of numerical data in (45a) and the subsequent discussion of (47b). Thus for $P_1 = 4$ atmospheres the superheating due the work $P_1 V_1'$ performed upon the contents $m' + \mu'$ of the clearance space v_1' so far exceeds the cooling influence of the low temperature T_2 of the portion μ' , that the temperature T_1'' is about 120° Fahr., or 56 degrees above T_1' .

Hence in (47b) the numerator must be greater than $(m' + \mu' + m'')T_1' = (m + \mu')T_1'$, whence T_1''' must be greater than T_1' .

The work is equal to

$$W_m = \frac{k}{k-1}(P_1 V_1' - P_2 V_2) + (P_2 - P_0)V_2 \\ + \frac{1}{k-1}(P_0 - P_2)v'. \quad (52)$$

§ 16. In order that the machine should work to the best advantage it is evidently necessary that the air should leave the cylinder at atmospheric pressure, that is, that P_2 should equal to P_0 . There ought then to exist a certain relation, between the volume of the compression cylinder $V_2 + v$, the pressure in the reservoir P_1 and the volume of the expansion cylinder $V_2 + v'$, which may be determined by the above equations. To fix the dimensions of a machine we may assume $V_2 + v$ and P_1 as given, and then deduce the value of $V_2 + v'$.

If we make $P_2 = P_0$, equations (50) and (51) will become

$$P_0(V_2 + v')^k = P_1 \left\{ (V_1' + v') \right. \\ \left. - \frac{1}{k} \frac{P_1 - P_0}{P_1} v' \right\}^k, \quad (52a)$$

and

$$P_1 V_1 = RmT_1,$$

whence

$$V_1 + v' = (V_0 + v) \left(\frac{P_0}{P_1} \right)^{\frac{k-1}{k}} \left\{ \frac{T_1'}{T_0} - \frac{1}{k} \right. \\ \left. - \frac{P_1 - P_0}{P_0} \frac{v'}{V_0 + v} + \frac{P_1}{P_0(V_0 - v)} \left(v' - \frac{T_1'}{T_1} v \right) \right\}. * \quad (53)$$

* The term $\frac{P}{P_0(V_0 - v)} \left(v' - \frac{T_1'}{T_1} v \right)$ is not given by Ledoux, as he probably considers equal per cents of clearance in both compression and expansion cylinders, in which case we have the approximate relation $v' = \frac{T_1'}{T_1} v$. This approximation is not admissible for all percentages of clearances.

It should be noted that V_0 in (53) is the volume $A_1'A_0$ in Fig. 1, p. 4. If then the volume A_2A_0 be taken as one cubic foot, which is the case in Table III., we have

$$V_0 = 1 + v \left(\frac{P_1}{P_0} \right)^{1/k} - v,$$

which substituted in (52a) gives (53) by considerable reduction.

A more convenient form for calculating the value of V_1 may, however, be obtained from equation (52a) as follows:

We have

$$V_1' = \frac{RmT_1'}{P_1}.$$

The work is

$$\left. \begin{aligned} W_m &= \frac{k}{k-1} (P_1 V_1' - P_0 V_2), \\ \text{or} \\ W_m &= \frac{m k c}{A} (T_1' - T_2). \end{aligned} \right\} \quad (54)$$

This value for the work is the same as found in § 7, where no waste space was allowed for; only the final temperature T_2 being greater for the same weight and pressure, the work of the air is less.

Substituting this in equation (52a) we have

$$V_2 + v' = \left(\frac{P_1}{P_0}\right)^{\frac{1}{k}} \left(\frac{RmT_1'}{P_1} + v' - \frac{1}{k} \frac{P_1 - P_0}{P_1} v' \right)$$

Hence

$$V_2 + v' - \left(\frac{P_1}{P_0}\right)^{\frac{1}{k}} \left(v' + \frac{1}{k} \frac{P_1 - P_0}{P_1} v' \right) = \left(\frac{P_1}{P_0}\right)^{\frac{1}{k}} \frac{RmT_1'}{P_1}.$$

From which

$$V_2 = \frac{\left(\frac{P_1}{P_0}\right)^{1/k} \frac{RmT_1'}{P_1}}{1 + \frac{v'}{V_2} \left[1 - \left(\frac{P_1}{P_0}\right)^{1/k} \left(1 - \frac{1}{k} \frac{P_1 - P_0}{P_1} \right) \right]} \quad (53a).$$

In which $\frac{v'}{V_2}$ is the proportion of clearance in the expansion cylinder.

The work of the resistance of the machine is then

$$\left. \begin{aligned} W_r - W_m &= \frac{k}{k-1} [P_1(V_1 - V_1') \\ &\quad - P_0(V_0 - V_2)] + \frac{k}{k-1} P_0 v \frac{V_0 - V_1}{V_1 + v}, \\ \text{or} \\ W_r - W_m &= \frac{m k c}{A} \\ &\quad (T_1 - T_1' - T_0 + T_2). \end{aligned} \right\} (55)$$

The negative heat produced is

$$Q = m k c (T_0 - T_2), \quad (56)$$

$$Q_1 - Q = A(W_r - W_m). \quad (57)$$

The useful effect of the machine is

$$u = A \frac{T_0 - T_2}{T_1 - T_1' - T_0 + T_2}, *$$

or

$$A \frac{T_0}{\frac{T_1 - T_1'}{T_1 - T_1'''} \cdot T_1 - T_0}. \quad (58)$$

* We have from eq. (51a) and (39) $\frac{T_1'''}{T_2} = \frac{T_1}{T_0}$. Hence $T_0 - T_2 = \frac{T_1 - T_1'''}{T_1} T_0$. Substituting these relations in the equation for u we obtain eq. (58).

As T_1''' is greater than T_1 , the useful effect is less than if there had been no waste space.

§ 17. Table III exhibits the results of a machine having a waste space of 4 per cent of the volume described by the pistons. The amount of air used being one cubic foot at 59° Fahr., and weighing .07653 lbs. In the cooler the air is brought to 64.4° Fahr.

By comparing these results with those of Table I, we see that the effect* of waste spaces is by no means to be neglected since it results in a loss of about 400 heat units for each theoretic horsepower per hour.

* If the clearance of the compressor was as great as four per cent. in practice, the effect of this clearance would be to require the volume $A_1'A_2$, Fig. 1, p. 4, for four atmospheres, to be about 11 per cent of the length of the compressing cylinder.

As a matter of fact, however, the clearance of compressors is not quite one per cent of the piston displacement. This would reduce the extra size of cylinder to about $\frac{2}{4}$ per cent, but the theoretical loss in economy would be the same as given above for four per cent clearance since by eq. (53a) the volume V_2 , and hence the temperature T_2 for a given weight m of air are independent of the compressor clearance.

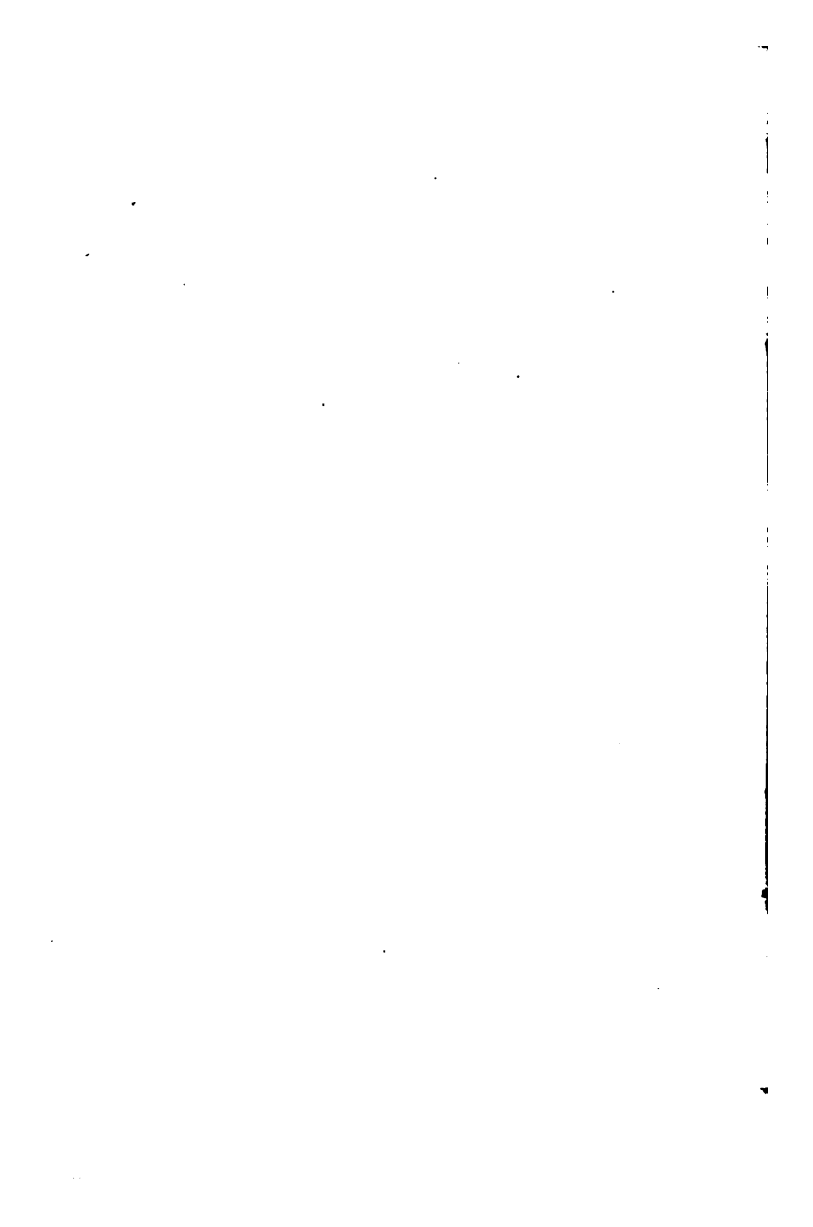
REFRIGERATING AIR FROM THE AIR SPACES INTO THE COMPRESSION CHAMBER, ASSUMING CLEAR PISTON DISPLACEMENT.

m = weight of air

$m = \text{weight of air}$			$W = \text{work due expansion}$	
Atmospheres. P_1	Temperature of air after adia- batic compression. t_1	Temperature of air as it leaves expansion cylinder. t_2	ICE MELTING CAPACITY PER CUBIC FT. OF AIR	CONDENSING
Deg. F.	Deg. F.	Deg. F.	Without friction Cub. Ft. of 12.2 / 2000.	With friction Cub. Ft. of 12.2 / 2000.
(1)	(2)	(3)	Ice melting capacity per cubic foot piston displacement. Cub. Ft. of 12.2 / 2000.	Ice melting capacity per cubic foot piston displacement. Cub. Ft. of 12.2 / 2000.
			Per cent. of piston displace- ment, assuming 300 F. range of temperature. $(t_1 - t_2) \div (300 - 80)$.	Per cent. of ice melting ca-
$1\frac{1}{2}$	123.9	+ 6	0.11	0.15
2	174.8	- 30	0.12	0.16
$2\frac{1}{2}$	217.3	- 50	0.13	0.17
3	254.1	- 70	0.14	0.18
$3\frac{1}{2}$	286.8	- 90	0.15	0.19
4	316.4	- 100	0.16	0.20
$4\frac{1}{2}$	343.4	- 110	0.17	0.21

adoux.

To fuel



§ 18. We can neutralize the influence of waste space by closing the outlet valve of cylinder B before the end of the stroke, so as to compress the air in this space; the stroke of the piston being exactly determined, the air in the waste space may be brought at the opening of the inlet valve to the temperature T_1' and the pressure $P_1'.$ *

In this case equations (34) and (43) apply without change.

During the period of expansion we have

$$P_2(V_2 + v')^k = P_1(V_1' + v')^k, \quad (59)$$

$$P_2(V_2 + v') = R(m + \mu')T_2, \quad (60)$$

$$P_1 v' = R\mu' T_1' \quad (61)$$

whence

$$\frac{T_2}{T_1'} = \left(\frac{V_1' + v'}{V_2 + v'} \right)^{k-1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}. \quad (62)$$

* This condition will not be exactly realized, unless the air at the commencement of compression is at the same pressure as the air at the end of expansion, for, if this is not the case, the air compressed in the clearance space will be at a different temperature than T_1' .

We must adjust the "cushioning" caused by the piston of the expansion cylinder so as to compress the air in the waste spaces from P_0 to P_1 , hence

$$\left. \begin{aligned} W_m = & \frac{k}{k-1}(P_1 V_1' - P_2 V_2) \\ & + (P_2 - P_0)V_2 + \frac{k}{k-1}P_0 v' \frac{V_0 - V_1}{V_1 + v} \\ & + \frac{1}{k-1}(P_0 - P_2)v', * \end{aligned} \right\} (63)$$

or

$$\begin{aligned} W_m = & \frac{mkc}{A}(T_1' - T_2) + (k-1)\frac{mc}{A}T_2 \\ & \left(1 - \frac{P_0}{P_2}\right) - (k-1)\frac{\mu'c}{A}\left(\frac{P_0}{P_2}T_2 - T_2'\right) \\ & + \frac{\mu'c}{A}(T_2' - T_2). \dagger \end{aligned}$$

T_2' being the temperature of the air in the cylinder at the moment compression commences before the end of the stroke.

* See foot-note p. 42.

† The term $\frac{\mu'c}{A}(T_2' - T_2)$ is not given by Ledoux.

We have then

$$\left. \begin{aligned} W_r - W_m &= \frac{k}{k-1} \\ &[P_1(V_1 - V_1') - P_0 V_0 + P_2 V_2] \\ &+ (P_0 - P_2)V_2 + \frac{k}{k-1}P_0(v - v') \\ &\frac{V_0 - V}{V_1 + v} - \frac{1}{k-1}(P_0 - P_2)v'. \end{aligned} \right\} \quad (64)$$

When the machine is well-regulated, the final pressure $P_2 = P_0$ and the equations (63) and (64) become

$$\begin{aligned} W_m &= \frac{k}{k-1}(P_1 V_1' - P_0 V_2) \\ &+ \frac{k}{k-1}P_0 v' \frac{V_0 - V_1}{V_1 - v}, \end{aligned} \quad (65)$$

or

$$W_m = \frac{m k c}{A}(T_1' - T_2),$$

and

$$\left. \begin{aligned} W_r - W_m &= \\ &\frac{k}{k-1}[P_1(V_1 - V_1') - P_0(V_0 - V_2)] \\ &+ \frac{k}{k-1}P_0(v - v') \frac{V_0 - V_1}{V_1 + v}. \end{aligned} \right\} \quad (66)$$

We have also

$$\frac{V_o + v}{V_1 + v} = \frac{V_o + v'}{V_1' + v'} \quad (67)$$

We see that in equation (66) the term relating to waste spaces disappears if we make $v = v'$. The equation then becomes

$$W_r - W_m =$$

$$\frac{k}{k-1} [P_1(V_1 - V_1') - P_o(V_o - V_o)].$$

The volume $V_o + v'$ is determined by means of equations (39), (41), and (67) when the pressure P_1 is known.

Reciprocally when V_o , v , V_1 , and v' , the dimensions of the machine, are known then V_1' is readily found, and consequently P_1 and T_1 , the pressure and temperature at the end of the stroke in cylinder B to insure the escape of the air at the atmospheric pressure.

Effect of Injecting an Excess of Water into the Compressor.

§ 19. It was remarked in § 5 that the efficiency of the machine could be no-

tably improved by cooling the air in the interior of the compressor cylinder.

This result can be accomplished, in part at least, if not completely, by means of a jet of water, such as is employed in air compressors.

We will proceed to calculate the work necessary for the compression in this particular case, neglecting the effect of waste spaces.

Let m be the weight of dry air occupying the volume V_0 . Let M represent the weight of water injected together with the amount of moisture into the air, and Mx the weight of the vapor at any instant.

The dilatation or compression of the mixture of the vapor and air is effected in such a manner as to satisfy the differential equation,

$$mcdt + M(dq + dx\rho) = -APdV, \quad (69)$$

which expresses the fact that variations in the internal heat of the mixture equal the variations of work accomplished.

We have also

$$dq = c_1 dt,$$

c_1 being the specific heat of water.

We may then write the differential equation

$$\begin{aligned} (mc + Mc_1)dt + A\rho dV &= - [Md(x\rho) \\ &+ A(P - p)dV] = - [Md(x\rho) \\ &+ A\frac{R}{V}mTdV], \text{ [by (70a)]} \end{aligned} \quad (69a)$$

p being the tension of the vapor, and P that of the mixture.

Then

$$x\rho = xr - A\rho xu,$$

$$d(x\rho) = d(xr) - A\rho d(xu) - Axudp,$$

and

$$dV = Md(xu).$$

$$\therefore d(x\rho) = d(xr) - \frac{A\rho dV}{M} - Axudp.$$

ρ is the latent heat of the vapor;

r is the heat of vaporization;

u is the increase of volume of a kilogram of water vaporized.

We know also that

$$Axu \frac{dp}{dt} = \frac{xr^*}{T}.$$

We have then

$$Md(x\rho) + ApdV = M \left[d(xr) - \frac{xr dt}{T} \right]$$

or

$$\frac{Md(x\rho) + ApdV}{T} = Md \left(\frac{xr}{T} \right).$$

By (69a) we then have

$$(mc + Mc_1) \frac{dt}{T} + ARm \frac{dV}{V} = -Md \left(\frac{xr}{T} \right).$$

Integrating between the limits T_1 and T_0 ,

$$(mc + Mc_1) l \frac{T_0}{T_1} + M \frac{x_0 r_0}{T_0} - M \frac{x_1 r_1}{T_1} + ARml \frac{V_0}{V_1} = 0, \quad (70)$$

$$Mx_0 = \frac{V_0}{u_0} \quad \text{and} \quad Mx_1 = \frac{V_1}{u_1};$$

$\frac{1}{u_0}$ and $\frac{1}{u_1}$ are very nearly the weights per

* Rankine's Steam Engine, art. 255.

cubic foot of the vapor under the pressures p_0 and p_1 .

We have, furthermore,

$$V_1 = Rm \frac{T_1}{P_1 - p_1} \quad (70a)$$

Equation (70) will give M when T_1 and T_0 are known.

$$AW_r = mc(T_1 - T_0) + M(q_1 - q_0 + x_1\rho_1 - x_0\rho_0) + A(P_1V_1 - P_0V_0)$$

or

$$AW_r = mkc(T_1 - T_0) + M(q_1 - q_0 + x_1r_1 - x_0r_0). \quad (71)$$

This equation gives the work of resistance when M is known.*

* The two equations (70) and (71), which express the relations between the volumes and the temperatures of a mixture of air and vapor, which is compressed or dilated, and which determine also the value of the work, are applicable to the Mekarski compressed air street-car motor.

In this machine the compressed air is reheated just before it is introduced into the expansion cylinder by being forced through water, having a temperature of 100° to 150°. The cylinder then contains air and saturated vapor, heated to a mean temperature of 100°.

The weight M of equations (70) and (71) is then the weight of the vapor contained in air, saturated at the temperature at which it leaves the hot water.

In M. Colladon's compressors, into which a spray of water is injected, the air being compressed to four atmospheres, the temperature T_1 does not rise above 122° F. ,* the external air being about 59° F.

* No indicator cards were taken off the Colladon compressor, it being *assumed* that the low temperature of the air at exit from the compressor greatly reduced the work of compression. It is now known, however, by experiments on other compressors, that a temperature at the exit as low as even 90° F. produces but *little* effect in reducing the *work of compression*. The fact seems to be that the cooling influence of the water occurs *after* the compression is complete.

In the case of the test of the Bell-Coleman air-refrigerating machine, reported by Prof. Schröter, air compressed to about 4 atmospheres was elevated in temperature only 34° F. , its temperature of exit from the compressor averaging about 80° F. Nevertheless the curve of compression is practically identical with the adiabatic. In the case of experiments made by Mr. F. T. Gause, at Hoboken, a spray was made by the collision of two streams of water striking each other at an angle, as in the case of Colladon's practice. Such spray injected into an 8×12 air-compressor under 200 lbs. pressure, so that the volume of water supplied was 1/500 part of the volume of air compressed (the volume of water in Colladin's practice was only about 1/1200 of the volume of air) failed to secure a lower value of the exponent α than 1.25. Such an exponent realizes about 1/3 of the theoretical saving possible if the compression curve was an isothermal line, i.e., followed the law of Mariotte. The compressor was run at 100 revs. per

We deduce then per cu. ft. of air compressed,

$$V_1 = 0.28429 \text{ cu. feet;}$$

$$M = .0357 \text{ lbs.;}$$

$$W_r = 3132 \text{ ft.-lbs.}$$

When the compression is effected without external cooling, we found in § 11 that the work of compression = 3614 foot-pounds, which shows a gain in the above process of about 13 per cent.

It remains to determine W_r for any pressure without any known value of T_1 .

When a certain volume of air is dilated or compressed, with or without the addition of heat, the relation of pressure to volume is expressed by the equation

$$PV^a = \text{a constant.}$$

$$\frac{V_1}{V_0} = \left(\frac{P_0 - p_0}{P_1 - p_1} \right)^{\frac{1}{a}} \quad (72)$$

minute, single-acting, and the highest air-pressure was 100 lbs. above the atmosphere. The exit temperature of the air was maintained at 90° F., either by the perfect spray injection or by the same quantity of water injected as a solid half-inch stream. In the latter case the exponent a averaged 1.86.

and

$$\frac{T_1}{T_0} = \left(\frac{P_1 - p_1}{P_0 - p_0} \right)^{\frac{a-1}{a}} \quad (73)$$

which gives

$$\frac{a-1}{a} = \frac{\log T_1 - \log T_0}{\log (P_1 - p_1) - \log (P_0 - p_0)}; \quad (74)$$

T_1 having been found by experiment, equation (74) gives a .

Making, in (74), $P_1 = 4$ atmospheres, $T_1 = 581.4$, and $T_0 = 518.4$ we find $a = 1.0912$.* a being thus determined, equation (73) will give T_1 . Only p_1 being a function of T_1 , the latter must be found by successive approximations.

Equation (70) gives

$$Mc_1 = 0.4343 \frac{\frac{V_0 r_0}{u_0 T_0} - \frac{V_1 r_1}{u_1 T_1}}{\log \frac{T_1}{T_0}} + 0.5888m.$$

r_0 , u_0 , r_1 , and u_1 are furnished by the tables.

* It is evident from the preceding foot-note, p. 65, that so low a value of a was not realized in the Coladon compressors which gave $t_1 = 122^\circ$ or $T_1 = 581.4$.

Finally we obtain W_r by equation (71). The saturated air in passing into the cooler is reduced in temperature from T_1 to T_1' , and a portion of the vapor is condensed. The weight of vapor remaining and introduced into the expansion cylinder is

$$\mu_1 = \frac{V_1'}{u_1'},$$

$\frac{1}{u_1'}$, being the weight per cu. ft. of the vapor corresponding to the temperature T_1' .

We will now calculate the cooling produced by the expansion and the work as explained in § 13.

§ 20. Table IV exhibits the results obtained from a cubic foot of air saturated at 59° , since the sides of the compressor cylinder are covered with water. The weight of the air is .07524 lbs.

An examination of this table and a comparison with the table of § 14 shows:*

* By virtue of facts stated in foot-note, page 65, the efficiency available by water injection should be considered to be about the same as in Tables I, II, and III.

REFRIGERATING 0.6 PER CENT OF ITS VOLUME. THE
ENTERS, COOLED IN THE CONDENSER TO 64.4°
CUBIC FOOT OF THE MIX-
= .07524 LBS.

$m = \text{weight}$

$W_m = \text{work due to expansion of a}$

P ₁ Atmospheres.		Temperature of air after adiabatic compression, <i>t</i> ₁		Temperature of air as it leaves expansion cylinder, <i>t</i> ₂		ICE-MELTING CAPACITY PER POUND OF COAL.*		CONDENSING V	
Atm.	Deg. F.	Deg. F.	With friction. Col. 13 + 142.2.	Without friction. Col. 14 + 3.	With friction. Col. 15 + 3.	Ice-melting capacity per cubic foot piston displacement, Col. 5 + (142.2 × 3000).		Per cubic foot piston displacement assuming 80° F. range of temperature, <i>Q</i> ₁ + (30 × 84).†	Per ton of ice-melting capacity. Col. 19 + Col. 18.
(1)	(2)	(3)	(15)	(16)	(17)	(18)	(19)	(20)	
2½	99.8	— 35.4	39.8	27.2	13.3	.00000591	.0111	1880	
3½	108.3	— 59.4	37.1	22.6	12.4	.00000744	.0136	1830	
3½	115.3	— 78.4	36.3	21.0	12.1	.00000866	.0156	1800	
4½	122.0	— 95.4	33.8	18.5	11.8	.00000969	.0174	1800	
4½	127.6	— 108.4	32.7	17.5	10.9	.00001055	.0189	1790	

Columns 14 to 21
Values for P₁ =
ice is formed during
The values for
 t_2 involves the cond
* Assuming 3 lb
+ Q_1 may be for

mitted because the final temperature is such
expansion is nearly completed, whereas the equa

$$+ .475 \frac{V_1}{u_1} + \left(\frac{V_1}{u_1} - x_1' \mu \right) r_1'.$$

The values give
other to within about 1/2 per cent.

To face pag

1st. That the injection of water into the interior of the compressor cylinder increases the efficiency 40 to 50 per cent.

2d. That the efficiency is at a maximum at a pressure of 2 1/2 atmospheres.

3d. That it diminishes, though slowly, as we vary from this pressure.

4th. That the quantity of snow or ice produced is not greater than that which comes from the moisture of the atmosphere.

The most favorable working pressure appears to be in this case nearly 4 atmospheres, since we obtain then a sufficiently good result (2.7 to 2.8 negative heat units per cu. ft. of air), with a relatively good performance of 4800 negative heat units per hour per horse-power.

Theoretically the injection of water into the compressor affords a great advantage. But it is possible that the water resulting from the condensation of vapor in the cooler does not all remain in the reservoir, but that a portion* is

* The water is in practice mechanically separated by the use of several receivers, or by diaphragms or de-

carried mechanically into cylinder B. The results indicated above for the efficiency would in such a case be considerably modified, and the increase in the quantity of frozen vapor would constitute in practice a grave inconvenience.

Experiment can alone decide this question.

Refrigerating Machines Employing Vapors of Volatile Liquids.

We have examined in the preceding pages nearly all the problems relating to the air machine. We will pass now to the study of the second class of machines, or those which transform motive force into negative heat by the employment of a liquefiable gas.

§ 21. The principle of these machines is the same as that of the kind described in the last chapter. The gas is compressed, then deprived of its heat, and

lector plates in a special receiver. Moisture is also precipitated from the air by chilling it while under pressure or before it reaches the expander.

finally caused to expand in such a manner as to lower its temperature. Only in this instance the abstraction of the heat which follows the compression has the effect to liquefy the gas, and it is the vaporization of the resulting liquid which produces the lowering of the temperature.

When a change of volume of a saturated vapor is made under constant pressure, the temperature remains constant. The addition or subtraction of heat, which produces the change of volume, is represented by an increase or a diminution of the quantity of liquid mixed with the vapor.

On the other hand, when vapors, even if saturated, are no longer in contact with their liquids, and receive an addition of heat, either through compression by a mechanical force, or from some external source of heat, they comport themselves nearly in the same way as permanent gases, and become superheated.

It results from this property, that refrigerating machines using a liquefiable

gas will afford results differing according to the method of working, and depending upon the state of the gas, whether it remains constantly saturated or is superheated during a part of the cycle of working.

§ 22. We will suppose first that the gas is constantly saturated, and will examine the conditions to be fulfilled under this hypothesis and the results that may be obtained.

Employing the notation of the preceding chapter, we will designate by m the weight of the gas employed, P_2 and T_2 the pressure and the absolute temperature of the cooled gas, P_1 and T_1' the pressure and the absolute temperature in the condenser.

The pressures P_2 and P_1 are determined by the temperatures T_2 and T_1' . These are the pressures of a saturated vapor at these temperatures, and are given in Regnault's tables.

The temperature of the condenser is determined beforehand by local conditions. Depending on the surface, the

interior of the condenser will exceed by 9° or 18° the temperature of the water furnished to the exterior. This latter will vary from about 52° F. the temperature of water from considerable depth below the surface, to about 95° F., the temperature of surface water in hot climates. The volatile liquid employed in the machine ought not at this temperature to have a tension above that which can be readily managed by the apparatus.

On the other hand, if the tension of the gas at the minimum temperature is too low, it becomes necessary to give to the compression cylinder large dimensions, in order that the weight of vapor compressed by a single stroke of the piston shall be sufficient to produce a notably useful effect.

These two conditions, to which may be added others, such as those depending on the greater or less facility of obtaining the liquid, upon the dangers incurred in its use, either from its inflammability or unhealthfulness, and finally upon its ac-

tion upon the metals, limit the choice to a small number of substances.

The gases or vapors available are: Sulphuric Ether, Sulphurous Oxide, Ammonia, Methylic Ether, and Carbonic Acid.

Table V., derived from Regnault, exhibits the tensions of the vapors of these four substances at different temperatures between -22° and $+104^{\circ}$. The original tables expressed the tensions in millimeters of mercury. To facilitate computation, the tensions are here given in *pounds per square inch*.

An inspection of the table shows at once that the use of ether does not readily lead to the production of low temperatures, because its pressure becomes then very feeble.

The ether machine is, however, abandoned. Ammonia on the contrary is well adapted to the production of low temperatures; but its elastic force is very great at temperatures from 59° to 86° which are readily produced in the condenser. It is not a good aid to the transformation

TABLE V.—PRESSURES AND BOILING POINTS
OF LIQUIDS AVAILABLE FOR USE IN
REFRIGERATING MACHINES.

Tempera- ture of Ebullition.	Tension of Vapor, in lbs. per sq. in., above Zero.					
Deg. Fahr.	Sul- phuric ether.	Sul- phur di- oxide.	Am- monia.	Methy- lic ether.	Car- bonic acid.	Pictet fluid.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
- 40	10.22
- 31	13.23
- 22	5.56	16.95	11.15
- 18	7.23	21.51	13.85	251.6
- 4	1.30	9.27	27.04	17.06	292.9	13.5
5	1.70	11.76	33.67	20.84	340.1	16.2
14	2.19	14.75	41.58	25.27	393.4	19.3
23	2.79	18.31	50.91	30.41	453.4	22.9
32	3.55	22.53	61.85	36.34	520.4	26.9
41	4.45	27.48	74.55	43.13	594.8	31.2
50	5.54	33.26	89.21	50.84	676.9	36.2
59	6.84	39.93	105.99	59.56	766.9	41.7
68	8.38	47.62	125.08	69.35	864.9	48.1
77	10.19	56.39	146.64	80.28	971.1	55.6
86	12.31	66.37	170.83	92.41	1085.6	64.1
95	14.76	77.64	197.83	1207.9	73.2
104	17.59	90.32	227.76	1338.2	82.9

Columns 2 to 6 have been deduced directly from the results given by Regnault. The figures given above do not agree precisely, in the case of carbonic acid, with the table given at the end of this work. This is due to the fact that in some cases the pressures in the table are calculated by a formula that is made different in form from that given by Regnault. Column 7 is derived from a curve of pressures and temperatures given in a work by R. Rudolff Gröbbs, entitled "Kompressions Kälte-Maschinen." The Pictet fluid is a mixture containing about 97 per cent of sulphur dioxide and 3 per cent of carbonic acid.

of mechanical force into heat, on account of the difficulty of maintaining tight joints in the apparatus, and of the influence of waste spaces at the high pressures.* Methylic ether yields low temperatures without attaining too great pressures at the temperature of the condenser. Finally, sulphur dioxide readily affords temperatures of $+14^{\circ}$ to $+5^{\circ}$, while its pressure is only 3 to 4 atmospheres at the ordinary temperature of the condenser. These two latter substances then lend themselves conveniently for the production of cold by means of mechanical force.

§ 23. Let c be the specific heat of the liquid employed.

q the quantity of heat necessary to raise 1 pound of the liquid from 32° to $T^{\circ} - 459.4^{\circ}$.

$$q = c(T - 459.4)$$

* American practice has reduced the clearance to less than one per cent and improvements in fittings now makes pressures of 200 lbs. per sq. inch as safe as were pressures of 60 lbs. ten years ago.

λ, r, ρ the, total heat, the heat of vaporization, and the internal latent heat of the vapor considered at the temperature $T^\circ - 459.4.^\circ *$

u , the increase of volume of one pound of liquid vaporizing at $T^\circ - 459.4.^\circ$

We have by definition

$$\lambda = r + q,$$

$$\rho = r - APu.$$

We will apply indices to these quantities similar to those which affect the letter T in designating the different absolute temperatures.

In order that the vapor be constantly saturated, it is necessary that the quantities of liquid and of vapor taken into the compressor at once be such that at the end of the compression all the liquid shall be vaporized and the vapor shall not be superheated.

If we let x_2' represent the proportion

* The quantity $r = \rho + APu$ is called by English writers the latent heat of evaporation. This is the quantity given by Regnault's experiments, and tabulated in D. K. Clark's Manual, as the latent heat.

of vapor contained in the mixture at the commencement of the inflow, the work of compression will be equal to the difference in the amount of internal heat of the mixture at the beginning and end of the compression, that is to say to

$$m(q_1' - q_2 + \rho_1' - x_2'\rho_2). * \quad (74a)$$

The work of the inflow into the condenser will be P_1V_1' , in which V_1' is the volume occupied by a weight m of the vapor at the end of the compression; and the work of the back pressure will be P_2V_2 , V_2 being the volume occupied by the weight mx_2' of vapor.

We have also

$$V_1' = m\left(u_1' + \frac{1}{\delta}\right),$$

and

$$V_2 = mx_2'\left(u_2 + \frac{1}{\delta}\right), \quad (75)$$

* This is the work between the compression curve and the line of zero back pressure. The work of outflow is not included. This formula holds good for those vapors that tend to superheat during compression, and must be slightly modified to fit those that condense—such as ether. See § 24, page 50.

δ being the weight of the liquid in lbs. per cu. ft. supposed constant.

We may neglect the fraction which is very small, and write

$$\begin{aligned} V_1' &= mu_1', \\ V_2 &= mx_2'u_2, \end{aligned}$$

from which we may get

$$mr_1' = m\rho_1' + AP_1V_1', \quad (75a)$$

and

$$mr_2 = m\rho_2 + AP_2V_2. \quad (75b)$$

The total work of the compression including the outflow is

$$AW_r = m(q_1' - q_2 + r_1' - x_2'r_2).^* \quad (76)$$

As the compression follows an adiabatic curve, the quantities q_1' , q_2 , r_1' , r_2 , T_1' and T_2 bear the following relation:†

$$\int_{T_2}^{T_1'} \frac{cdt}{T} = \frac{x_2'r_2}{T_2} - \frac{r_1'}{T_1'}$$

* The total work of compression including outflow = the value given in (74a) + $P_1V_1' - P_2V_2'$. Substituting in this the value given in equations (75a) and (75b) we obtain (76).

† This is the fundamental equation of adiabatic expansion of a saturated vapor. Rankine St. Eng., Art. 284; Röntgen's Thermodynamics, Chap. XXIII.; Wood's Thermodynamics, Art. 112.

or more simply,

$$\frac{r_1'}{T_1'} + cl \frac{T_1'}{T_2} = \frac{x_2' r_2}{T_2}. \quad (77)$$

Equation (77) will give the quantity x_2' . Consequently equation (75), when we know m , furnishes the volume V_2 , that the piston should describe during the aspiration in order that all the liquid shall be vaporized at the end of the compression; or, inversely, the weight m may be found if V_2 be given

The vapor flows into the condenser where it is liquefied.

The heat absorbed by the water of the condenser is

$$Q_1 = m r_1'. \quad (78)$$

The liquid then passes into the expansion cylinder where it is vaporized, producing work till it attains the pressure P_2 and the temperature T_2 of the refrigerant. At the end of the expansion, the weight of vapor in the mixture is $m x_2$.

The work, including the counter-pressure, and neglecting the work of introducing the liquid $p_1 \frac{m}{\delta}$, which is very small, is

$$AW_m = m(q_1' - q_2 - x_2 r_2), \quad (79)$$

and the equation of the adiabatic curve is

$$\frac{x_2 r_2}{T_2} = cl \frac{T_1'}{T_2}, \quad (80)$$

which determines x_2 .

The quantity of heat Q necessary to bring the mixture whose weight is $m(1 - x_2)$ of liquid and mx_2 of vapor to its primitive condition, in which $m(1 - x_2')$ is the weight of the liquid and mx_2' is the weight of the vapor, is,

$$Q = m(x_2' - x_2)r_2,*$$

or by reason of equations (77) and (80)

$$Q = \frac{T_2}{T_1'} m r_1'. \quad (81)$$

* It is assumed that there is no superheating of the vapor in the refrigerating coils.

The work expended is $W_r - W_m$ and we have

$$\begin{aligned} A(W_r - W_m) &= m[r_1' - (x_2' - x_2)r_2] \\ &= Q_1 - Q_2 \quad (82) \end{aligned}$$

The "efficiency" of the machine is

$$\frac{Q}{W_r - W_m} = \frac{AQ}{Q_1 - Q_2} = A \cdot \frac{T_2}{T_1' - T_2}, \quad (83)$$

a result already found in section 3, and which is identical with that at which we arrived in the case of permanent or non-liquefiable gases.

§ 24. We will now take a numerical example, and consider the dimensions of the cylinders to be so regulated that a final temperature of $+5^\circ$ F. is obtained, the temperature of the condenser being $+64.4^\circ$ F., and the volume of gas taken into the compressor at each stroke, $V_1 =$ one cubic foot.

The resolution of the above equations supposes a knowledge of the values of r , q , c , and u , or APu . They have been determined directly by Regnault for sulphuric ether, but not for sulphur

dioxide, ammonia and methylic ether. Availing ourselves of the experiments of Regnault* upon the compressibility of gases, we have been able to determine these quantities for sulphur dioxide and ammonia, and to prepare tables giving results for every nine degrees from -22° to $+104^{\circ}$.

The method of calculating these tables will be found in a note at the end of this essay.

For sulphur dioxide we find

$$\begin{array}{ll}
 t_2 = 5^{\circ} \text{ or } T_2 = 464.4, & t_1' = 64.4 \text{ or } T_1' = 523.8, \\
 r_2 = 170.82, & r_1' = 156.76, \\
 AP_2 u_2 = 14.26, & AP_1 u_1' = 15.40, \\
 q_2 = -9.79, & q_1' = 11.80, \\
 u_2 = 6.50, & u_1' = 1.88.
 \end{array}$$

The table of § 22 gives $\frac{P_2}{144} = 11.76$
 and $\frac{P_1}{144} = 44.33$ lbs. per sq. in.

* The quantity determined by Regnault, which is here used, is the value of PV at 8.1° C. See appendix and deductions, § 27.

Making the calculations indicated by the equations (77) and (80) we find

$$x_2' = 93.29 \text{ per cent,}$$

$$x_2 = 11.90 \text{ per cent.}$$

Equations (75) gives

$$m = .1645 \text{ lbs.}$$

Equations (76) and (79) give

$$AW_r = 3.13 \text{ B. T. U. ;}$$

whence

$$W_r = 2417 \text{ ft. lbs.}$$

$$AW_m = .21;$$

whence

$$W_m = 161 \text{ ft. lbs.}$$

Finally equations (78) and (81) give

$$Q_1 = 25.80 \text{ B. T. U.}$$

$$Q = 22.88 \quad "$$

Thus the volume described by the piston of the compression cylinder being one cubic foot, .1645 lbs. of sulphur dioxide working between $+5^\circ$ and $+64.4^\circ$ produce 22.88 negative heat units. To effect this it is necessary to introduce

into the compressor cylinder at each stroke a mixture of liquid and gas of which the proportion should be 93.29 per cent of gas and 6.71 per cent of liquid.

AMMONIA.

From Table XIX we have for ammonia

$$\begin{aligned} t_2 &= 5^\circ, & t_1' &= 64.4, \\ \frac{P_2}{144} &= 33.67 \text{ lbs.}, & \frac{P_1'}{144} &= 117.44, \\ r_2 &= 580.56, & r_1' &= 543.03, \\ AP_2 u_2 &= 51.86, & AP_1 u_1' &= 57.12, \\ u_2 &= 8.27, & u_1' &= 2.63, \\ q_2 &= -26.42, & q_1' &= 33.68. \end{aligned}$$

The mean specific heat of the liquid at 32° is $c = 1.0058$.

By means of these given values we find

$$\begin{aligned} x_2' &= 92.62 \text{ per cent}, \\ x_2 &= 9.68 \text{ per cent}, \\ m &= .1303 \text{ lbs.}, \\ AW_r &= 8.53 \text{ B. T. U.}, W_r = 6591 \text{ ft. lbs.}, \\ AW_m &= .51 \text{ B. T. U.}, W_m = 391 \text{ ft. lbs.}, \\ Q_1 &= 70.78 \text{ B. T. U.}, \\ Q &= 62.75 \text{ B. T. U.} \end{aligned}$$

Thus .1303 lbs. of ammonia working between the same limits of 64.4° and $+5^{\circ}$, and with the same dimensions of compressor cylinder as sulphur dioxide, furnish 62.75 negative heat units per hour.

CARBONIC ACID.

From Table XX we have for carbonic acid *

$t_2 = 5^{\circ}$	$t_1' = 64.4$
$\frac{P}{144} = 342 \text{ lbs.}$	$\frac{P_1'}{144} = 826$
$r_2 = 121.50$	$r_1' = 67.93$
$AP_2 u_2 = 15.50$	$AP_1' u_1' = 9.94$
$u_2 = .2435$	$u_1' = .0650$
$q_2 = -20.92$	$q_1' = 35.80$

The mean specific heat of the liquid from 64.4° to 5° as derived from Table XX is

$$c = .955.$$

* The case of carbonic acid has been added by the revisers.

Employing the above values we find:

$$x_1' = 93.52 \text{ per cent } x_1 = 43.93 \text{ p. c.}$$

$$m = 4.119 \text{ lbs.}$$

$$AW_r = 45.40 \text{ B.T.U. } W_r = 35,090 \text{ ft.-lbs.}$$

$$AW_m = 13.78 \text{ B.T.U. } W_m = 10,650 \text{ ft.-lbs.}$$

$$Q_1 = 279.80 \text{ B. T. U.}$$

$$Q = 248.18 \text{ B. T. U.}$$

ETHER.

We will now consider ether. The vapor of ether, unlike steam, superheats during expansion and condenses during compression. An ether machine ought, therefore, to work so that only vapor is introduced into the compressor cylinder, and not a mixture of liquid and vapor. At the end of the compression a part of the vapor becomes condensed.

We shall then have $x_1' = 1$, and the equations above found become:

$$V_1' = mx_1' \left(u_1' + \frac{1}{\delta} \right),$$

$$V_2' = m \left(u_2 + \frac{1}{\delta} \right),$$

$$\frac{x_1' r_1'}{T_1'} = \frac{r_2}{T_2} - c \cdot l \frac{T_1'}{T_2},$$

$$\frac{x_2 r_2}{T_2} = c l \frac{T_1'}{T_2},$$

$$Q_1 = m x_1' r_1',$$

$$Q = m(1 - x_2) r_2,$$

$$AW_r = m(q_1' - q_2 + x_1' r_1' - r_2),$$

$$AW_m = m(q_1' - q_2 - x_2 r_2).$$

The empirical formulas established by Regnault for the vapor of ether are

$$r = 171.24 - .04873t - .000473t^2,$$

$$APu = 11.78 + .0756t - .000752t^2,$$

$$q = - 16.76 + .51849t + .0001644t^2,$$

and we deduce:

$$\text{for } t = 5^\circ \quad \text{and} \quad t' = 64.4,$$

$$\frac{P_2}{144} = 1.70 \text{ lbs.} \quad \frac{P_2'}{144} = 7.76,$$

$$r_2 = 170.99, \quad r_1' = 166.14,$$

$$AP_2 u_2 = 12.14, \quad AP_1 u_1' = 13.53,$$

$$u_2 = 38.37,$$

$$q_2 = - 14.16, \quad q_1' = 17.31,$$

$$c = .5299,$$

and we have

$$\delta = .736.$$

Performing the calculations indicated, we find,

$$\begin{aligned}
 x_2 &= 17.32, \\
 x_2' &= 100, \\
 x_1' &= 95.97, \\
 m &= .0260, \\
 Q_1 &= 4.15, \\
 Q &= 3.68, \\
 AW_r &= .52, & W_r &= 401 \text{ ft.-lbs.}, \\
 AW_m &= .05, & W_m &= 37 \text{ ft.-lbs.}
 \end{aligned}$$

The same machine, working between 5° and $+64.4^\circ$, will therefore give the following cooling effect per cubic foot of compressor-piston displacement—

Carbonic acid...	248.18	negative heat units.
Ammonia.....	62.75	“ “
Sulphur dioxide.	22.88	“ “
Sulphuric ether..	3.68	“ “

With each substance the useful effect is .0101 thermal units per ft.-lb. of compression work, since all the substances are confined to a Carnot cycle between the temperatures $+5^\circ$ and $+64.4^\circ$.

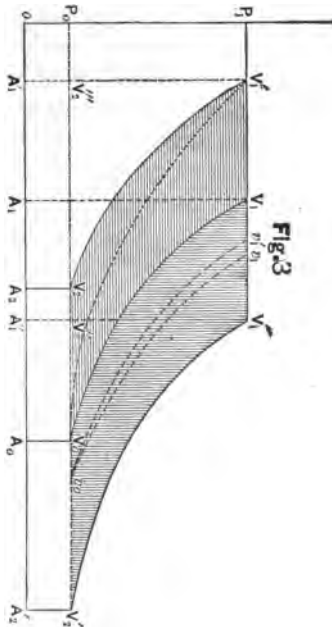
§ 25. Since the positive work W_m , except in the case of carbonic acid, is small

compared with the negative work W_r , we can, without great loss of power, simplify the machine by suppressing the expansion cylinder and replacing it by a simple cock so regulated as to deliver into the cooler a weight of fluid precisely equal to the amount admitted to the compressor to obtain the required cooling effect.

The cycle of operations is not reversible when a simple cock is used, and therefore does not fulfill the conditions of a Carnot cycle. We shall have $\frac{Q}{AW_r} = \frac{Q}{Q_1 - Q}$, but the proportion $\frac{Q}{Q_1 - Q}$ will be less than $\frac{T_2}{T_1' - T_2}$, and hence the efficiency is less.

This manner of working is represented in the diagram, Fig. 3, by replacing the adiabatic line $V_1'V_2$ by the two right lines $V_1'V_2'''$ and $V_2'''V_2''$. V_2'' is situated to the right of the point V_2 . The quantity Q proportioned to $V_2''V_2$ is therefore less than the quantity Q of the pre-

ceding case which was proportioned to V, V_0 , and the quantity $Q_1 - Q$ will be



The equations (76), (77), and (78) remain unchanged.

The weight m of the liquid under the pressure P_1 and the temperature T_1' passing suddenly into the refrigerating coils, a part of the liquid is vaporized; the temperature of the liquid becomes T_2 and the pressure P_2 . The quantity x_2 of liquid, which is vaporized, is given by the equation

$$m(q_2 - q_1' + x_2\rho_2) + AP_2V_2' - A(P_1 - P_2) \frac{m}{\delta} = 0,$$

which shows that the variation of internal heat $m(q_2 - q_1' + x_2\rho_2)$ is equal to the exterior work accomplished;

$$- AP_2V_2' + A(P_1 - P_2) \frac{m}{\delta};$$

V_2' being the volume occupied by the weight mx_2 of vapor after the passage of the mixture into the refrigerating coils.

We have

$$V_2' = mx_2 \left(u_2 + \frac{1}{\delta} \right).$$

If we neglect the very small quantity

$$AP_1 \frac{m}{\delta}$$

the preceding equation becomes

$$x_1 r_1 = q_1' - q_1. \quad (84)$$

The quantity Q is still given by the equation

$$Q = m (x_2' - x_2) r_2,$$

or by reason of eq. (76)

$$Q = m r_1' - A W_r = Q_1 - A W_r,$$

from whence the useful effect is

$$\frac{Q}{W_r} = A \frac{Q}{Q_1 - Q}. \quad (85)$$

The useful effect will be less than for the Carnot cycle because the value of x_2 given by eq. (84) is always greater than that given by eq. (80). Consequently the value of Q will be less in the second case than in the first, and hence the ratio $\frac{Q}{Q_1 - Q}$ will be less.

In applying equations (84) and (85) to the same limiting temperatures, $+ 5^\circ$

and 64.4° , as in § 24, we find for sulphur dioxide

$$x_2 = 12.64 \text{ per cent,}$$

$$Q = 22.68,$$

and the useful effect = .0094 heat units per ft.-lb. of work of compression $W_r - W_m$. For ammonia:

$$x_2 = 10.35 \text{ per cent,}$$

$$Q = 62.25,$$

and the useful effect = .0094 heat units per ft.-lb.

For sulphuric ether,

$$x_2 = 18.46 \text{ per cent,}$$

$$Q = 3.45,$$

useful effect = .0086 heat units per ft.-lb. of work.

Finally, for carbonic acid:

$$x_2 = 46.68 \text{ per cent,}$$

$$Q = 234.38,$$

$$\text{useful effect} = .0067.*$$

* It is seen that the loss of efficiency due to the use of the expansion cock is 7 per cent for sulphur dioxide and ammonia, 15 per cent for ether, and 33 per cent for carbonic acid.

§ 26. In order to realize either the cycle of Carnot or the non-reversible cycle indicated above, it is necessary, when we employ a liquefiable gas which superheats under compression, to introduce into the compressor cylinder at each aspiration a mixture of liquid and vapor in such proportions that it shall all be in the state of gas at the end of the compression. The preceding equations assume this to be accomplished.

We can, however, devise no means* of exactly realizing this condition in practice. So we content ourselves, when employing freezing machines that use a liquefiable gas, with introducing the gas into the compressor without any mixture of liquid. It follows therefore, that the gas superheats during compression with sulphur dioxide, ammonia, and carbonic acid.

* Although there is no practical means of exactly adjusting the mixture of vapor and liquid, certain ammonia plants are operated with a surplus of liquid present during compression, so that superheating is prevented. This practice is known as the "Cold System" of compression.

It is clear that under these conditions we augment the range of temperature between T_1 of the gas arriving in the condenser, and T_2 of the vapor in the refrigerating coils, and consequently increase the cooling effect of a given weight of substance. The efficiency is, however, reduced by the superheating, as will be evident from the following argument.

Referring again to Fig. 3, it is seen that we start with a volume v_0 greater than V_0 of the preceding case, compress the vapor to the volume v_1 following the adiabatic curve v_0v_1 of the superheated gas; cool it from the temperature T_1 to the temperature T_1' corresponding to its liquefaction under the pressure P_1 . It is then passed into the refrigerant, either producing work and describing the adiabatic curve V'_1V_1 or by means of a cock, in which case the volume, after entering the refrigerant is P_0V_1'' , and we lose the work of the expansion cylinder $V_2V_1'P_1P_0$.

The quantity of negative heat gained, if superheating is allowed to take place during compression, is represented by

the length V_0v_0 and the increase of resistant work by the area $V_0V_1v_1v_0$.

Tracing from the point v_0 , the adiabatic curve of the saturated vapor, the point v_1' will be to the left of v_1 .

If the compressed vapor follows the adiabatic v_0v_1' , the performance $\frac{Q'}{Q_1' - Q'}$ will be equal to the performance $\frac{Q}{Q_1 - Q}$ of the Carnot cycle $V_0V_1V_1'V_2$.

But as the compression follows the line v_0v_1 , we see that for the same quantity Q' of obtainable negative heat, the quantity $Q_1 - Q'$ would be greater by a quantity proportional to the area $v_0v_1v_1'$.

We can say, then, that, *à priori*, the useful effect of freezing machines working so as to superheat the gas is less than that of machines that work without superheating.

The difference* is small, as we shall see later.

* By the examples, §§ 30-34, it appears that the difference in economy is about 5 per cent for SO_2 , about 6 per cent for ammonia, and about 11 per cent for CO_2 , as compared with the use of an expansion cock without

§ 27. We will now establish the thermodynamic formulæ for the condition that we introduce into the cylinder during aspiration only gas, and in such a state as to superheat during compression.

A certain volume V_2 of gas being under pressure P_2 and temperature T_2 , it is required to find its volume V_1 and its temperature T_1 when it shall have attained the pressure P_1 of the condenser.

If liquefiable gases behaved like permanent gases, it would suffice to use the equations (1) to (6), which were established in § 10 for the compression of air.

But the researches of Regnault on the compressibility of gases have established the fact that when near the liquefying point these bodies are far from following the laws of Mariotte and Gay-Lussac, upon which the formulas for air are founded.

Zeuner has given (*Théorie Mécanique*

superheating; or 12 per cent, 13 and 41 per cent, respectively, as compared with a Carnot cycle, involving the use of an expansion cylinder, as per examples, § 24.

de la Chaleur) the result of his researches upon superheated steam.

He found the following relation to exist between the pressure P in lbs. per sq. ft., the volume of the unit of weight, or one pound (specific volume), v , and the absolute temperature T ,

$$Pv = BT - CP^n, * \quad (86)$$

in which C and n are constants to be determined by experiment

$$B = \frac{c_p n}{A}, \quad (87)$$

c_p being the specific heat of the vapor under constant pressure, which is constant according to Regnault.

If we make $k = 4/3$, $B = 92.84$, and $C = 938.9$, we find that this formula furnishes, for the specific volume of steam, numbers which agree remarkably well with the results of experiment.†

Zeuner does not offer this relation as

* See remark, p. 170, on origin of (86).

† $n = \frac{k-1}{k} = .25$.



rigorously exact, but as giving much better results than the formula

$$Pv = RT,$$

which applies to permanent gases.

Liquefiable gases being nothing but superheated vapors, we will employ equation (86), established for superheated steam, but will determine the constants in each case, employing the results of Regnault's experiments upon the dilatation and compression of gases.

If we call α the coefficient of dilatation of the gas under atmospheric pressure, and a temperature of 32° Fahr., it is easy to see that eq. (86) gives

$$\alpha = \frac{1}{491.4 - \frac{C}{B} 2117^n} *$$

and

$$2117v_0 = 491.4B - C. 2117^n; \quad (88)$$

whence

$$2117v_0\alpha = B, \quad (89)$$

* This value of α is simply the value of $\frac{dv}{v_0 dt}$ from (86), which is differentiated considering P as a constant equal to 2117 lbs. per sq. ft.

an equation which gives B when we know the coefficient of dilatation and specific volume v , at 32° and atmospheric pressure.

If the relation (87) were exact, it would suffice with equations (88) and (89) for determining B , C , and n . But the numbers thus obtained do not coincide, at least in the case of sulphur dioxide and ammonia, with the results obtained by Regnault. Instead therefore of using equation (87) we will determine n by the results found by Regnault for the product PV .*

Regnault gives values of PV for sulphur dioxide at the temperature of 35.06° , and at 46.58° for ammonia, and for pressures varying from 14 to 23 lbs. per sq. inch for sulphur dioxide, and from 13 to 28 lbs. per sq. inch for ammonia. We can deduce from these the ratio of the volumes at the high and low pressures, and thus obtain two values of PV in Zeuner's equation. From the density of

* The following method of determining the constants has been re-written by the revisers so as to give the several steps in greater detail.

the gas we obtain the volume v_0 at 32° and under atmospheric pressure, which together with the coefficient of dilatation enables us to determine two values of $\frac{PV}{m}$ for the temperature at which Regnault's experiments were made. We have from (86)

$$PV = mBT - mCP^n, \quad (90)$$

which, together with equation (89), will furnish B, C, and n , in the following manner :

For sulphur dioxide

$$\alpha = 0.0021682 \text{ per degree Fahr.};$$

$$v_0 = 5.513.*$$

Introducing these values in (89), we obtain

$$B = 25.305.$$

For atmospheric pressure and $T_0' = 494.5$,

* This corresponds to the density given by Berzelius.

$$\frac{V_o'}{m} = 5.514(1 + .002168 \times 3.06) = 5.55,$$

$$\frac{P_o V_o'}{m} = 5.55 \times 2117 = 11749.$$

For $P = 3347$ lbs. per sq. ft., or 23.2 lbs. per sq. in., and $T = 494.5$,

$$\frac{PV}{m} = 11749 \div 1.015^* = 11575.$$

Introducing the values of B , $\frac{PV}{m}$ and $\frac{P_o V_o'}{m}$, in (90), we obtain two equations in which C and n are the unknown quantities. Solving these equations, we have

$$C = 24.54,$$

$$\text{and} \quad n = .449.$$

Introducing these constants into equation (86) we can obtain for Pv values which coincide in a satisfactory manner with Regnault's results.

* The ratio 1.015 is given by Regnault's table of the values of PV .

These values are slightly less than Regnault's for pressures between 14.7 lbs. and 23.25 lbs., and a little larger for pressures lying beyond these limits on either side.

For ammonia we unfortunately do not know the coefficient of dilatation; it was not determined by Regnault. As this gas is near its liquefying point at 32° we will assume its coefficient to be about the same as that of sulphur dioxide and cyanogen, which is 0.002167. In the absence of exact values determined by experiment it is clear that results obtained under the above assumption can be regarded as approximative only.

We have $v_0 = 20.79$, and Regnault's tables give

$$\frac{PV}{m} = 44607 \text{ for } T = 506.0^{\circ} \text{ and } P = 3997 \\ \text{lbs. per sq. ft., or } 27.76 \text{ lbs. per sq. in.}$$

We then deduce

$$B = 95.31, \quad C = 157.1,^* \quad n = 0.378.$$

* This constant has been reduced in the ratio of 43.71 to 26.28 from the value given by Ledoux, which was in

§ 28. It remains now to find the equation of the adiabatic curve of a superheated vapor, of which the pressure, the specific volume, and the temperature are related as follows:

$$pv = BT - Cp^{*.*} \quad (90a)$$

Calling Q the quantity of heat furnished to a body, U its internal work, and supposing the external pressure is always equal to the expansive force, the fundamental equation of the mechanical theory of heat is,

$$dQ = A(dU + pdv);$$

error to this extent. For the argument leading to this alteration of the value of C , see article by Messrs. Jacobus and Reisenberger, *Stevens' Indicator*, Oct. 1890. Since this article was written the examination of a similar argument, by Zeuner, in his *Technische Thermodynamik*, Vol. II., Article 31, shows that he had found Ledoux's value of C in error also, but in the ratio of 43.71 to 29.78.

* Equations in which the volumes are indicated by small v 's are per unit of weight, and those in which large V 's are used are for any weight m . P and p are used to represent the same quantities, the small p being employed to save space in making deductions.

and as U is a function of p and v , we have

$$dQ = A \left\{ \frac{dU}{dp} dp + \left(\frac{dU}{dv} + p \right) dv \right\}.$$

Assuming

$$\frac{dU}{dp} = X \quad \text{and} \quad \frac{dU}{dv} + p = Y,$$

and since dU is an exact differential,

$$\frac{dX}{dv} = \frac{dY}{dp} - 1. * \quad (90b)$$

We also have

$$dQ = A(Xdp + Ydv). \quad (91)$$

We know that $\frac{1}{T}$ is the factor of integra-

$$* \frac{AdX}{dv} = \frac{Ad^2U}{dpdv},$$

$$\frac{AdY}{dp} = \frac{Ad^2U}{dvdp} + A,$$

or

$$\frac{AdX}{dv} - \frac{AdY}{dp} = -A,$$

or

$$\frac{dX}{dv} - \frac{dY}{dp} = -1.$$

bility of the function $Xdp + Ydv$, and hence

$$dQ = AT\left(\frac{X}{T}dp + \frac{Y}{T}dv\right).$$

We may now deduce from the general equations of thermodynamics

$$T = Y\frac{dt}{dp} - X\frac{dt}{dv}.* \quad (92)$$

* Since dQ multiplied by $\frac{1}{T}$ is a complete differential (see general transformation of equations of thermodynamics, Arts. 62-69 and 118, "McCullough on the Mechanical Theory of Heat") we have, from (91),

$$\frac{dQ}{T} = \frac{A}{T}(Xdp + Ydv),$$

and we may write

$$\frac{d}{dv}\left(\frac{A}{T}X\right) = \frac{d}{dp}\left(\frac{A}{T}Y\right),$$

or

$$\frac{AcX}{Tdv} - \frac{AdT}{T^2dv}X = \frac{A}{T}\frac{dY}{dp} - \frac{AdT}{T^2dp}Y;$$

whence

$$X\frac{dT}{dv} - Y\frac{dT}{dp} = T\left(\frac{dX}{dv} - \frac{dY}{dp}\right).$$

But by (90b) the second member of this is $-T$; hence since $T = t + 461$, and $dT = dt$,

$$Y\frac{dt}{dp} - X\frac{dt}{dv} = T.$$

We have also, by virtue of equation (86),

$$\frac{dt}{dp} = \frac{v}{B} + \frac{nCp^{n-1}}{B} \quad (92a)$$

and

$$\frac{dt}{dv} = \frac{p}{B}. \quad (92b)$$

If we suppose that the pressure remains constant, $dp = 0$, and eq. (91) gives

$$dQ_p = AYdv.$$

But $dQ_p = c_p dt$, calling c_p the specific heat at constant pressure, which we suppose constant and which is known. We have then

$$Y = \frac{c_p}{A} \frac{dt}{dv} = \frac{c_p}{A} \times \frac{p}{B}, \quad (92c)$$

and from eq. (92)

$$X = -\frac{B'T}{p} + \frac{c_p}{AB}(v + nCp^{n-1}), \quad (92d)$$

which by (90a) gives

$$X = -(v + Cp^{n-1}) + \frac{c_p}{AB}(v + nCp^{n-1}).$$

Substituting X and Y from (92c) and (92d) in (91), we have

$$dQ = A \left\{ \frac{c_p}{AB} (p dv + v dp) - v dp + \left(\frac{nc_p}{AB} - 1 \right) C p^{n-1} dp \right\}. \quad (93)$$

For the equation of an adiabatic curve, it is necessary to make $dQ = 0$. We have then,

$$\left(\frac{c_p}{AB} - 1 \right) d.pv + p dv + \left(\frac{nc_p}{AB} - 1 \right) C p^{n-1} dp = 0 \quad (94)$$

Equation (94)* may be reduced to

$$\frac{c_p}{AB} \frac{dt}{T} = \frac{dp}{p},$$

* Eq. (94) may be written

$$\frac{Cp}{AB} (p dv + v dp + n C p^{n-1} dp) = (pv + C p^n) \frac{dp}{p}$$

or

$$\frac{Cp}{AB} \times \frac{p dv + v dp + n C p^{n-1} dp}{pv + C p^n} = \frac{dp}{p}. \quad (94a)$$

By substituting the partial differential coefficients given in (92a) and (92b), in the expression for the complete differential

$$dt = \left(\frac{dt}{dp} \right) dp + \left(\frac{dt}{dv} \right) dv,$$

and integrating

$$\frac{c_p}{AB} lT = lp + \text{const.},$$

or finally

$$\left(\frac{T}{T_0}\right)^{\frac{cp}{AB}} = \frac{p}{p_0}, \quad (95)$$

an equation analogous to equation (4), which we found for air.

Replacing T by this value in equation (90a) we get finally for the equation of the adiabatic curve,

$$pv = BT_0 \left(\frac{p}{p_0}\right)^{\frac{AB}{c_p}} - Cp^n. \quad (96)$$

it appears that

$$dt = \frac{1}{B}(p dv + v dp + n Cp^{n-1} dp);$$

also by (86),

$$\frac{1}{T} = B \frac{1}{pv + Cp^n}.$$

Hence (94a) may be written

$$\frac{Cp}{AB} \times \frac{dt}{T} = \frac{dp}{p}. \quad (94b)$$

For superheated steam Zeuner finds $\frac{AB}{c_p} = n$, so that (96) is then equivalent to $pv^n = \text{a constant}$, as for permanent gases.*

From eq. (94) we have the work of compression

$$\int_{v_1}^{v_0} p dv = \int -dW =$$

$$\left(1 - \frac{c_p}{AB}\right) d.pv + \left(1 - \frac{nc_p}{AB}\right) Cp^{n-1} dp,$$

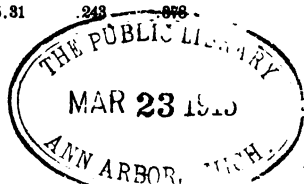
whence

$$W = \left(\frac{c_p}{AB} - 1\right)(pv - p_0 v_0) +$$

$$\left(\frac{c_p}{AB} - \frac{1}{n}\right) C(p^n - p_0^n); \quad (97)$$

* This relation does not hold for sulphur dioxide and ammonia, in which we have

	B	$\frac{AB}{c_p}$	n
Sulphur Dioxide	25.30	.212	.449
Ammonia	95.31	.243	.678



or by (96),

$$W = \left(\frac{c_p}{AB} - 1 \right) B(T - T_0) + C \left(1 - \frac{1}{n} \right) (p^n - p_0^n), \quad (98)$$

and .

$$W = \left(\frac{c_p}{AB} - 1 \right) B(T - T_0) + C p_0^n \left(1 - \frac{1}{n} \right) \left\{ \left(\frac{T}{T_0} \right)^{\frac{n c_p}{AB}} - 1 \right\}. \quad (99)$$

§ 29. We can now establish the equations relating to the compression of a liquefiable gas in a cylinder. A weight m of gas occupying the volume V_2 at the temperature T_2 , and under the pressure P_2 , is compressed until the pressure is P_1 of the condenser. The temperature T_1 at the end of the compression will be given by the equation (95).

$$T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{\frac{AB}{c_p}} \quad (100)$$

and the work of compression, including* the ejection of the gas, is

$$W_r = \frac{c_p}{AB}(P_1 V_1 - P_2 V_2) + m\left(\frac{c_p}{AB} - \frac{1}{n}\right)C(P_1^n - P_2^n), \quad (101)$$

or

$$W_r = \frac{mc_p}{A}(T_1 - T_2) - \frac{mCP_2^n}{n} \left\{ \left(\frac{T_1}{T_2}\right)^{\frac{nc_p}{AB}} - 1 \right\}. \quad (101a)$$

* Equations (97)–(99) give the area $A_0 V_0 V_1 A_1$ (Fig. 8) $= \int_{V_1}^{V_0} p dv$, whereas W_r is the area

$$V_0 V_1 P_1 P_0 = m \int_{V_1}^{V_0} p dv + P_1 V_1 - P_0 V_0.$$

Also (97)–(99) are for unit of weight, and W_r is for any weight m . Introducing these modifications in (97), we obtain (101).

Also introducing value of PV from (90) in (101) we obtain

$$W_r = \frac{mc_p}{A}(T_1 - T_2) - \frac{mCP_2^n}{n} \left[\left(\frac{P_1}{P_2}\right)^n - 1 \right]$$

which by (100) gives (101a).

m is given by the equation

$$m = \frac{P_2 V_2}{BT_2 - CP_2} = \frac{V_2}{u_2 + \frac{1}{\delta}} \quad (102)$$

the final volume

$$V_1 = V_2 \frac{P_2}{P_1} \cdot \frac{BT_1 - CP_1}{BT_2 - CP_2}.$$

After compression we cool the gas in the condenser under constant pressure. The volume V_1 becomes V_1' at the moment the temperature becomes T_1' ; since the gas is liquefied we have

$$V_1' = V_2 \frac{P_2}{P_1} \cdot \frac{BT_1' - CP_1}{BT_2 - CP_2},$$

and the quantity of heat removed from the condenser is

$$Q_1 = mc_p(T_1 - T_1') + mr_1'. \quad (103)$$

The volume occupied by the liquid is

$$v_1 = \frac{m}{\delta},$$

δ being the density of the liquid supposed constant.

The liquid is then passed into the refrigerant without producing work.

The quantity mx_2 of gas which vaporizes while the pressure passes from P_1 to P_2 and the temperature from T_1' to T_2 is, by equation (84),

$$mx_2r_2 = m(q_1' - q_2).$$

The quantity of negative heat obtained is

$$Q = m(1 - x_2)r_2$$

or

$$Q = m(\lambda_2 - q_1'), \quad (104)$$

and we have

$$Q_1 - Q = mc_p(T_1 - T_1') + m(r_1' + q_1' - r_2 - q_2)$$

or

$$Q_1 - Q = mc_p(T_1 - T_1') + m(\lambda_1' - \lambda_2).$$

We can now verify the equality

$$Q_1 - Q = AW_r$$

which requires that we establish the relation

$$\lambda_1' - \lambda_2 = c_p(T_1' - T_2) - \frac{AC}{n}(P_1^n - P_2^n).*$$

Referring to the fundamental equation

$$dQ = AdU + APdv,$$

and making $dQ = 0$, it becomes

$$mdU = -mPdV = -dW,$$

and consequently †

$$U_1 - U_2 = \left(\frac{c_p}{AB} - 1\right)B(T_1 - T_2) + C\left(1 - \frac{1}{n}\right)(P_1^n - P_2^n). \quad (105)$$

We have furthermore by definition

$$\lambda = AU + APv, \ddagger \quad (105a)$$

* This is the equivalent of the last equation in footnote, p. 113.

† This is equivalent of equation (98).

‡ In (105a) put $v = \frac{V}{m}$. In (105) substitute T_1 and T_2 from (90). Combine results and obtain (105b).

an equation which signifies that the total heat of the vapor at t° is equal to the internal heat AU augmented by the thermal equivalent of the work of vaporization and dilatation.

We have then

$$\lambda_1 - \lambda_2 = c_p(T_1 - T_2) - \frac{AC}{n}(P_1^n - P_2^n). \quad (105b)$$

This equation is applicable to a superheated vapor above its point of saturation.

It applies also at the point of saturation; we have then

$$\lambda_1' - \lambda_2 = c_p(T_1' - T_2) - \frac{AC}{n}(P_1^n - P_2^n), \quad (106)$$

which verifies the equation

$$Q_1 - Q = AW_r.$$

Equation (105) can be written

$$U - U_0 = \left(\frac{c_p}{AB} - 1 \right) (Pv - P_0 v_0) + \left(\frac{c_p}{AB} - \frac{1}{n} \right) C (P^n - P_0^n).$$

If we make $\frac{c_p}{AB} - \frac{1}{n} = 0$, the equation becomes

$$U = U_0 + \frac{1-n}{n}(Pv - P_0v_0).$$

Under this form it expresses Hirn's law of superheated vapors, and may be thus expressed: From the point of condensation to the point at which the superheated vapor possesses the same properties as the permanent gases, the product $p v$ remains constant, while the internal work remains the same.

But the equation

$$\frac{c_p}{AB} = \frac{1}{n}$$

is not verified * for the cases of the two liquefiable gases which we have studied, and consequently we cannot apply to them the law of Hirn.

§ 30. We will now take a numerical example and suppose, as in the preceding case, that a cubic foot of gas is admitted to the compressor at the temperature of

* See p. 111.

+ 5° under a pressure corresponding to this temperature, and that it is compressed until its tension is that of the condenser, and that the temperature of this latter is, in the interior, + 64.4°.

SULPHUR DIOXIDE.

Equation (102) gives

$$m = \frac{1}{6.50 + \frac{1}{62.4 \times 1.42}} = .1535 \text{ lb.}$$

Equation (100) gives, making

$$c_p = 0.15438$$

according to Regnault, and

$$\frac{AB}{c_p} = 0.212;$$

$$T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{0.212} = 616.0, \text{ or } t_1 = 156.6.$$

Equations (103) and (104) give

$$Q_1 = 26.25 \text{ B. T. U.};$$

$$Q = 22.91 \quad "$$

whence $AW_r = Q_1 - Q = 3.34 \text{ B. T. U.},$

and $W_r = 2582 \text{ ft.-lbs.},$

and the useful effect = .00887 or 17,563 heat units per hour per horse-power.

In a double-acting engine working at high velocity we estimate the resistances at about 15 per cent of the power expended.*

$1.15W_r = 2969$, and the useful effect becomes .00772 or 15,286 heat units per hour per horse-power.†

This performance is double that of the machine working with dry air between the same limits of temperature. This difference shows, not that the air is theoretically a less efficient agent in the production of cold, but that to produce the same useful effect, the air machine having much larger dimensions‡ than the liquefi-

* This factor varies from 10 to 25 per cent in machines of good construction.

† The ice-melting capacity in tons per cubic foot of piston displacement is .0000806, and the number of gallons of condensing water per ton, assuming 80° Fahr. range of temperature, is 1306. The ice-melting capacity per pound of coal, assuming 3 lbs. of coal per hour per horse-power, is 41.2 lbs., neglecting friction, and 35.8 lbs., including friction.

‡ Another view of the cause of the theoretical superiority of vapor machines is as follows :

For equal efficiency the air machine must afford a temperature of - 5° F. as the lowest degree afforded

able gas machines will experience proportionally greater loss through resistances.

by the expansion. By Table I this temperature will be seen to afford 1.164 thermal units of useful effect, and if this be taken as unity it will be found that the work of compression is 1.25 and the work of expansion 1.1 times as much. Each of these amounts of work wastes 8 per cent of itself in friction. Hence the aggregate of this waste is 18.8 per cent of the useful refrigerating effect. But the work of the steam-engine is only $1.25 - 1.1 = 0.15$ of the useful effect without friction being considered.

Hence the friction of the mechanism increases the expenditure of motive power $\frac{18.8}{15} = 1.25$ times. Therefore the refrigeration per unit of motive power is only $\frac{1}{1 + 1.25} = \frac{4}{9}$ as much as for zero friction. Now for vapors we have, by § 24:—

Work to compress both SO^2 and $\text{NH}^3 = 0.14$ of useful effect, and the loss by friction at 15 per cent of these amounts is 0.021 of the useful effect.

Hence the refrigeration per unit of motive power is $\frac{0.14}{0.14 + 0.021} = 7/8$ of its value for zero friction for both NH^3 and SO^2 . Therefore the air machine has its theoretical effect reduced $\frac{1}{8} = 2\frac{1}{4}$ times by the friction of its mechanism, or the loss is $\frac{1}{8}$ of the refrigerating effect, whereas the vapor machines lose but $1/8$ of their effect by friction.

More generally, the air machine loses more of its theoretical effect by friction than vapor machines, because the heat represented by the work of compression of a given weight of air is about 1.70 times greater in proportion to the refrigerative effect than in the case of ammonia and sulphur dioxide. In the practical use of air machines there is a considerable

§ 31. Generally with sulphur dioxide we do not get as low a temperature as 5° .

The opening of the cock which leads from the condenser to the refrigerating coils is so regulated that the pressure in the latter is about $9/10$ of an atmosphere, which corresponds to a temperature of about 9.66° .

$$P_2 \div 144 = 13.23 \text{ lbs.}; \quad t_2 = 9.66^{\circ}.$$

For these values Table XVIII gives

$$r_2 = 169.75;$$

$$q_2 = -8.10;$$

$$u_2 = 5.85 \text{ cu. ft.};$$

$$m = \frac{1}{u_2 + \frac{1}{62.4 \times 1.42}} = .1706;$$

and by means of equations (100), (102), (103), and (104) of § 29 it is easy to calculate T_1 , Q_1 , Q , and W_r .

The results of these calculations are recorded in Table VI, which gives the refrigerating effect, the work required, and the useful effect per cubic foot of

loss by the wire-drawing or fall of pressure between the compression and expansion cylinders. *Vide Practical Examples of Air Machines, Introduction.*

**REFRIGERATING POWER OF ONE
SIMPLE COCK TO 18.23
THIS PRESSURE**

t_1'	$P_1 + 144$	t_1	Q_1	Q	W_r
Temperature corresponding to pressure of vapor in condenser.	Absolute pressure in condenser.	Temperature at end of compression.	Heat carried away from condenser.	Negative thermal units or useful refrigerating effect.	Work of compression without friction.
Deg. F.	Lbs. per sq. in.	Deg. F.	B.T.U.	B.T.U.	Ft. lbs.
(1)	(2)	(3)	(4)	(5)	(6)
59	39.98	183.4	28.93	25.90	2340
68	47.61	156.0	28.91	25.84	2761
77	56.89	178.4	28.89	24.78	3180
86	66.86	200.8	28.87	24.22	3596
95	77.64	223.2	28.84	23.66	4006
104	90.31	245.4	28.81	23.09	4416

Col. 18 + 1440.

Colt

sulphur dioxide, supposing the apparatus regulated so that the temperature corresponding to the pressure in the refrigerating coils is 9.66° , and that the temperature corresponding to the pressure of the interior of the condenser varies from 59° to 104° .

We see that the useful effect diminishes more than one-half when the temperature of the interior of the condenser rises from 59° to 104° .

The figures of the last column do not nearly represent the number of heat units really produced and utilized. It is necessary to take into account the loss occasioned by the pipes, the waste spaces in the cylinder, loss of time in opening of the valves, the leakage around the piston and valves, the reheating by the external air, and finally, when ice is being made, the quantity of the ice melted in removing the blocks from their moulds.

It requires about 180 heat units to congeal to 20° a pound of water taken at 60° or 65° . Manufacturers estimate that practically the sulphur dioxide apparatus using water at 55° or 60° pro-

duces 56 lbs. of ice,* or about 10,000 heat

* In the commercial manufacture of ice about 7 lbs. are produced per pound of coal. This includes the fuel used for reboiling the water, which, together with that wasted by the pumps and lost by radiation, amounts to a considerable portion of that used by the engine.

The figures given in Table VI are higher than those obtained in practice, because the effect of superheating of the gas during admission to the cylinder is not considered. This superheating may cause an increase of work of about 25 per cent. There are other losses due to superheating the gas at the brine tank, and in the pipe leading from the brine tank to the compressor, so that in actual practice a sulphur dioxide machine, working under the conditions given in line three of the table, will give about 22 lbs. of ice-melting capacity per pound of coal, which is about 60 per cent of the theoretical amount neglecting friction, or 70 per cent including friction. The following tests, selected from those made by Prof. Schroter on a Pictet ice machine having a compression cylinder 11.3 inches bore and 24.4 inches stroke, show the relation between the theoretical and actual ice-melting capacity.

No. of Test.	Temp. in degrees Fahr. corresponding to pressure of vapor.		Ice-melting capacity per pound of coal, assuming three pounds per hour per horse- power.		
	Con- denser.	Suction.	Theoret- ical fric- tion ¹ in- cluded.	Actual.	Per cent loss due to cylin- der super- heating, or difference be- tween cols. 4 and 5.
11	77.3	28.5	41.3	33.1	19.9
12	76.2	14.4	31.2	24.1	22.8
13	75.2	-2.5	23.0	17.5	23.9
14	80.6	-15.9	16.6	10.1	39.2

¹ Friction taken at figure observed in the test, which ranged from 23 to 26 per cent of the work of the steam cylinder.

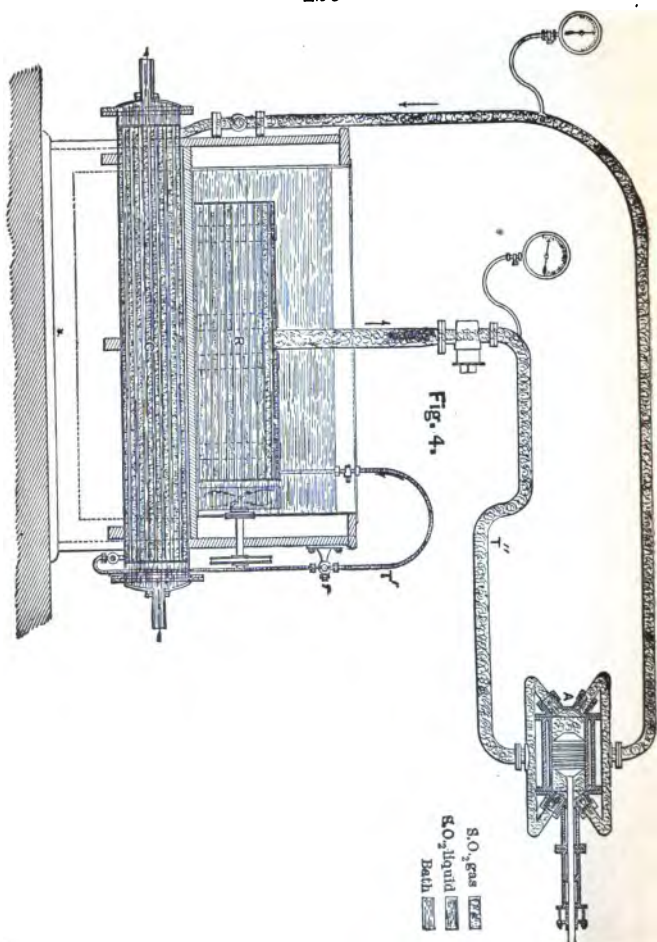
units, per hour per horse-power, measured on the driving shaft, which is about 55 per cent of the theoretic useful effect indicated in Table VI.

§ 32. Figs. 4 and 5 represent the Pictet machine from a design furnished by the inventor. It has a double-acting compression cylinder with four valves. The cylinder is furnished with a jacket, within which a current of cold water is made to circulate.

The gas is compressed to a tension corresponding to the temperature of the water employed for cooling, generally 25 to 30 lbs. per sq. in. above the atmosphere; then it is discharged by the pipe T into the condenser C, where it is liquefied.

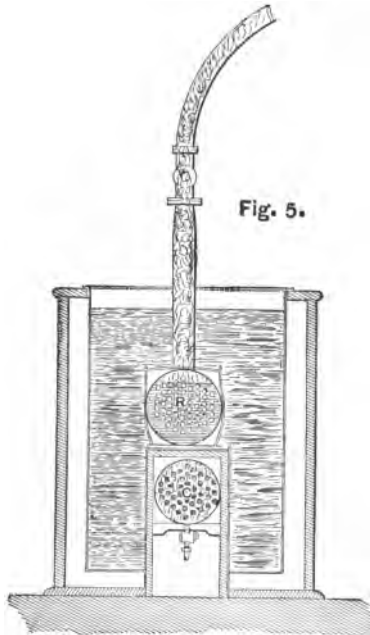
This condenser is like the surface condensers of marine engines. It has a surface of about 65 sq. ft. for 100,000 *theoretic* heat units per hour, or 130 sq. ft. for 100,000 *effective* heat units per hour, measured by the ice produced.

The quantity of water employed depends upon the difference of temperature



to be allowed between the inside and outside of the condenser.

If this difference is to be 9° , each pound



of water releases 9 heat units, and the weight of water in pounds to be employed

will be for 100 theoretic heat units produced

$$\frac{Q_1 100}{9Q} = 11.1 \frac{Q_1}{Q},$$

which would require for the example of § 31 and for a temperature of 68° in the condenser, 12.7 lbs.*

The liquid dioxide passes into the cooler R by the pipe T', the supply being regulated by the cock *r* so that the pressure shall be 9/10 of an atmosphere in the refrigerating coils and 3 atmospheres in the condenser. If the outlet by the cock be diminished the pressure is lowered in the cooler, and the temperature is also lowered; but the useful effect diminishes, since for the same volume described by the compressor piston less weight of gas is used. We have in this machine, therefore, the same facilities for varying the useful effect as in the air machines.

The refrigerating coils are arranged like the condenser. Their surface is 79 sq. ft. for each 100,000 theoretic negative heat

* In the additions to Ledoux's tables the range of temperature of the condensing water is taken as 30° F.

units produced per hour. It is immersed in an incongealable bath formed of a solution of calcium chloride.

The temperature corresponding to the pressure of the sulphur dioxide in the refrigerating coils of the cooler is 10.4° , and that of the bath 19.4° . In this bath are immersed the tanks or moulds within which the water is frozen.

Finally, the sulphur dioxide returns to the compressor cylinder by the pipe T''.

The dioxide may be employed continuously so long as no air is permitted to enter the joints. Any leakage might lead to the production of the trioxide, and possibly sulphuric acid, which would lead to injury to machine. Exceptional care is required in maintaining tight joints.

AMMONIA.

§ 33. Some experiments with an ammonia machine have not yielded very good results; but the want of success seems to have resulted rather from an imperfect action of the surface of the cooler than from any inherent defect in the gas itself. Ammonia gas possesses

the advantage of affording about three times the useful effect of sulphur dioxide for the same volume described by the piston. But this advantage is balanced by the inconvenience of higher pressures, and consequently more leakage, etc.*

The specific heat of ammonia gas at constant pressure is 0.50836,† so that

$$\frac{AB}{c_p} = .243.$$

The results of the calculations for ammonia are given in Table VII.‡

* The perfection of ammonia apparatus now renders it so convenient and reliable that no practical advantage results from the lower pressures afforded by sulphur dioxide.

† This was given by Regnault for atmospheric pressure only. By an experiment on a large scale on a 75-ton refrigerating machine, it has been determined that the probable specific heat of superheated ammonia gas at 162 lbs per square inch absolute pressure is .532. Paper 432, Volume XII., Transactions of American Society of Mechanical Engineers.

‡ The theoretical results for ammonia are higher than the actual, for the same reasons that have been stated for sulphur dioxide. See foot-note, page 124. In the case of ammonia the action of the cylinder walls in superheating the entering vapor has been determined experimentally by Prof. Denton, and the amount found to agree with that indicated by theory. In these experiments the ammonia circulated in a 75-ton refriger-

**PERFORMANCE OF AMMONIA COMPRESSOR.
TEMPERATURE OF CONDENSING WATER.**

Temperature corresponding to pressure of vapor in refrigerating coils t_2	Absolute pressure in refrigerating coils. $P_2 + 144$	Latent heat at temperature corresponding to P_2 . r_2	Increase of volume during evaporation in the refrigerating coils. u_2	Heat of liquid at temperature corresponding to P_2 . q_2	Temperature of gas at end of compression.* t_1	CONDENSING WATER.	
						Weight of gas compressed. temperature. m	Per minute per ton of ice-melting capacity in 24 hours. Col. 19 + 1440.
Deg. F.	Lbs. per sq. in.	B. T. U.	Cu. ft.	B. T. U.	Deg. F.	Lbs. s.	Gals.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(20)
9.66	37.76	577.96	7.50	-21.94	158.9	.1329 0	0.89
5.00	33.67	580.56	8.27	-26.42	170.1	.1206 0	0.91
-22.00	16.95	594.86	15.64	-51.35	241.3	.0639 0	0.98

* The temperature at the end of compression given in the value of B given by Ledoux. The difference between it and the error of the quantities involved.

CARBONIC ACID.*

At atmospheric pressure and 32° Fahr. the density of CO₂ is 1.529 and its co-

ating machine was measured directly by means of a special meter, so that in addition to determining the effect of superheating, the latent heats can be calculated at the suction and condenser pressure. See Appendix, page 185. The following is a comparison of the theoretical ice-melting capacity with that obtained in several of Prof. Schroter's tests on a Linde machine having a compression cylinder 9.9 inches bore and 16.5 inches stroke, and also in tests by Prof. Denton on an ammonia compression machine having two single-acting compression cylinders 12 inches bore and 30 inches stroke.

No. of Test.	Temp. in degrees Fahr. correspond- ing to pressure of vapor.		Ice-melting capacity per pound of coal, assuming three pounds per hour per horse- power.			
	Con- denser.	Suction.	Theoret- ical fric- tion ¹ in- cluded.	Actual.	Per cent of loss due to cylinder su- perheating.	
Schroeter {	1	72.3	26.6	50.4	40.6	19.4
	2	70.5	14.3	37.6	30.0	20.2
	3	69.2	0.5	29.4	22.0	25.2
	4	68.5	-11.8	22.8	16.1	29.4
Denton {	24	84.2	15.0	27.4	24.2	11.7
	26	82.7	-3.2	21.6	17.5	19.0
	25	84.6	-10.8	18.8	14.5	22.9

¹ Friction taken at figures observed in the tests which range from 14 to 20 per cent of the work of the steam cylinder.

* Added by the revisers.

efficient of dilatation .002067. Introducing these values in equation (89) we have

$$B = \frac{2117 \times .002067}{.08073 \times 1.529} = 35.45.$$

The mean value of c_p given by Regnault from -22° to 50° Fahr. is .184; hence

$$\frac{AB}{c_p} = .249.$$

CASE 1. Let the temperature of the cooler be 5° and that of the condenser 64.4° ; then

$$T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{.249} = 578.5^\circ \quad \text{or} \quad t_1 = 119.1^\circ.$$

From Table XX we have for $t_2 = 5^\circ$

$$m = 3.853 \text{ lbs.}$$

Taking c_p as .200, which is the mean from 64.4° to 119.1° , in equation (103), we have

$$Q_1 = 303.6 \text{ B. T. U.}$$

Equation (104) gives

$$Q = 249.6 \text{ B. T. U.},$$

and

$$AW_r = Q_1 - Q = 54.0 \text{ B. T. U.},$$

and

$$W_r = 41,740 \text{ ft.-lbs.}$$

Including friction, we have

$$1.15W_r = 48,000 \text{ ft.-lbs.}$$

The useful effect, neglecting friction, is .00598 B. T. U. per ft.-lb. of work expended, and, including friction, .0052 B. T. U.

The ice-melting capacity in tons per cubic foot of piston displacement is .000878.

The number of gallons of condensing water per ton of ice-melting capacity, assuming a range of 30° Fahr. in the condensing water, is 1380. The ice melting capacity per pound of coal, assuming 3 lbs. of coal per hour per horse-power, is 27.7 lbs. neglecting friction, and 24.1 lbs. including friction.

CASE 2. Let the temperature of the

cooler be -22° Fahr. and that of the condenser 64.4° Fahr. We then have

$$t_1 = 155.7^{\circ},$$

$$m = 2.321 \text{ lbs.},$$

$$Q_1 = 200.0 \text{ B. T. U.}$$

$$Q = 145.2 \quad "$$

$$AW_r = 54.8 \quad "$$

$$W_r = 42,350 \text{ ft.-lbs.},$$

useful effect, neglecting friction, $= .00343$,

" " including " $= .00298$.

Ice-melting capacity per cu. ft. piston displacement $= .00051$ tons. Condensing water per ton of ice-melting capacity $= 1570$ gallons. Ice-melting capacity per pound of coal, neglecting friction, $= 15.9$ lbs.; including friction, $= 13.8$ lbs.

VAPOR OF WATER.

For a numerical example of this gas in "Vacuum Machines," see Addendum, page 186.

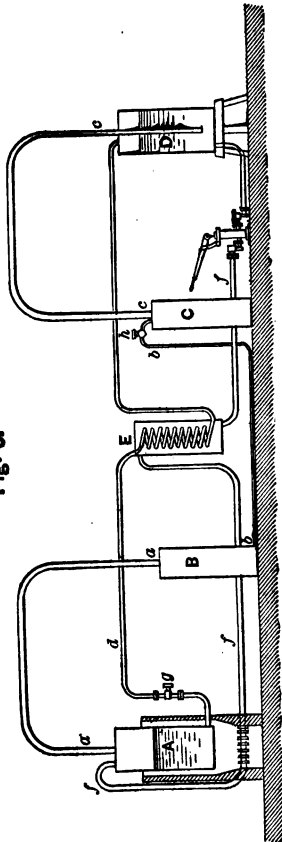
CHAPTER IV.

MACHINES EMPLOYING CHEMICAL ACTION.

§ 34. It remains to discuss the ice-making machines which employ chemical affinity in their mode of action, and of which the ammonia machine of M. Carré is the type.

Fig. 6 exhibits the disposition of the parts of this apparatus. It consists of a generator, A which contains a concentrated solution of ammonia in water; this generator is heated either directly by a fire, as shown in the figure, or indirectly by pipes leading from a steam boiler. The condenser B communicates with the upper part of the generator by the tube *aa*; it is cooled externally by a current of cold water. The cooler or brine tank C is so constructed as to utilize the cold produced; the upper part of it is in communication with the lower part of the con-

Fig. 6.



denser by means of the tube *bb*. The details of the construction are not shown in the figure. An absorption chamber **D** is filled with a weak solution of ammonia; the tube *cc* puts this chamber in communication with the cooling tank **C**.

The absorption chamber communicates with the boiler by two tubes. One, *dd*, leads from the bottom of the generator to the top of the chamber **D**; the other, *ff*, leads from the bottom of **D** to the top of the generator. Upon the pipe *ff* is mounted a little pump, whose use is to force the liquid from the absorption chamber, where the pressure is maintained at about one atmosphere, into the generator, where the pressure is from 8 to 12 atmospheres.

An interchange of heat is accomplished by means of **E** which is attached to the pipes *ff* and *dd* in a manner that will be easily comprehended by an inspection of the figure.

To work the apparatus the ammonia solution in the generator is first heated. This releases the gas from the solution,

and the pressure rises. When it reaches the tension of the saturated gas at the temperature of the condenser there is a liquefaction of the gas, and also of a small amount of steam. By means of the cock *h* the flow of the liquefied gas into the refrigerating coils contained in the cooler *C* is regulated. It is here vaporized by absorbing the heat from the substance placed there to be cooled. As fast as it is vaporized it is absorbed by the weak solution in *D*. The small quantity of watery vapor is carried along mechanically.

Under the influence of the heat in the boiler *A*, the solution is unequally saturated, the stronger solution being uppermost.

The weaker portion is conveyed by the pipe *dd'* into the chamber *D*, the flow being regulated by the cock *g*, while the pump sends an equal quantity of strong solution from *D* back to the boiler. While these exchanges are brought about in the solutions, there is also an exchange of temperatures at *E* whereby the weak liquid arrives cold in the absorption

chamber, and the strong solution is heated before its delivery to the generator.

The working of the apparatus depends upon the adjustment and regulation of the cocks *g* and *h*, and of the pump; by means of these the pressure is varied, and consequently the temperature in the cooler C may be controlled.

It is seen that the working is similar to that of the machines described in the preceding chapters. The chamber D fills the office of aspirator, and the generator A plays the part of compressor.

The mechanical force producing exhaustion is here replaced by the affinity of water for ammonia gas; and the mechanical force required for compression is replaced by the heat which severs this affinity and sets the gas at liberty. We see then in advance that a greater part of the equations already established for liquefiable gas machines will apply to those employing chemical action.

§ 35. We will assume at first that, under the influence of the heat applied to the generator, ammonia gas only is driven

off, and no steam. We will assume a certain weight of the gas to enter the generator in a state of solution; being heated, it will be separated from the water, requiring a certain quantity of heat, which we will call Q' . Then, being conducted to the condenser, it will be cooled and then liquefied, and will impart to the water surrounding the coils a quantity of heat Q_1 . In the refrigerating coils it is evaporated, borrowing from the exterior a quantity of heat Q ; it is next absorbed by the liquid in the chamber D, disengaging a certain amount of heat to the liquid (which may be deducted from the total amount required in the generator);* and, finally, it is reconveyed to the generator, where it arrives in its original condition. By reason of the exchange of temperature effected at E all the heat of the weak solution going out of the generator is restored to the strong solution entering it, so that the changes of temperature in the water are effected without expenditure of heat.

* See next foot-note, p. 141.

In the complete cycle, if we neglect the small amount of work performed by the pump, and the heating and cooling due to contact with the air, it is clear that all the heat from external sources, being Q' from the generator and Q from the cooler, will be equal to the amount Q_1 carried away by the water of the condenser.

We have then

$$Q' = Q_1 - Q,$$

and the useful effect will be expressed by

$$A \frac{Q}{Q_1 - Q}, *$$

which is identical with that found for the machines depending on mechanical action.

* This expression applies only when it is assumed, as is here the case, that the heat disengaged by the solution of the gas in the absorber may all be utilized to heat the mixture delivered to the generator. Thus far practice has only succeeded in using a temperature in the absorber about 70° greater than that in the condenser. Consequently the greater proportion of the chemically disengaged heat in the absorber must be abstracted by cold water and wasted; so that practically the above expression does not apply, but it is a generalization useful in apprehending the general principles of the subject.

Q' being equal to the quantity of heat which it is necessary to expend in order to produce the quantity Q of refrigerating effect, then $Q_1 - Q$ has the same value as the quantity AW_r , the thermal equivalent of the mechanical work expended in the machines previously discussed, to produce this same quantity Q of negative heat units. We proceed to show that between the same limits of temperature in the condenser and refrigerating coils, and for the same value of Q , the quantity Q' in this class of machines is nearly equal to the quantity AW_r .

We arrive then at this remarkable result: that in all the ice machines, when they work between the same limits of temperature, the theoretic quantity of refrigerating effect produced is exactly the same for each heat unit expended, whether it is directly produced by chemical action, or indirectly under the form of mechanical work.

But as a heat unit represented by 772.9*

* This is deduced from the French equivalent 424, and

foot-pounds requires, in the best heat motors, an expenditure of at least 10 heat units in the fire, it would seem that the chemical machines possess a considerable advantage over all the others, since in these latter the heat is employed directly, and not under the expensive form of mechanical work. Practically,* however, this advantage is much less than seems to result from the above calculations, as we will proceed to show.

§ 36. We will assume the hypothesis mentioned in the beginning of the preceding section, and determine the quantities Q' , Q_1 , and Q in terms of the temperatures, the pressures, and weights of the gas employed.

We will preserve the notations of the previous chapter; T_1 being the absolute

is employed throughout the work in order to make the figures correspond to those given by Ledoux.

* The heat wasted in maintaining the absorber at 180° , the highest temperature yet attained, causes a loss roughly equivalent to the heat wasted in the exhaust of a steam-engine using 8 lbs. of coal per hour per horse-power. The efficiency of the compression and absorption machines is therefore nearly equal in practice. For practical results, see Introduction.

temperature of the gas as it enters the condenser, T_1' its absolute temperature in the condenser, and T_2 the absolute temperature in the refrigerating coils.

Let m be the weight of the gas considered, occupying the volume V_2 at the temperature T_2 , and under the pressure P_2 at its entrance into the absorption chamber.

Let AU be the internal heat at the temperature T ; q_e the heat necessary to raise a pound of water from 32° to t° .

After the gas has been absorbed by the water the absolute temperature of the mixture will be T_2' .

During the process of absorption of the gas there is an amount of external work accomplished equal to $P_2(V_2 - w)$, w being the volume of water.

The difference in internal heat before and after this operation is equal to this external work. We have then

$$q_{e2}' + mAU_2' - q_{e2} - mAU_2 = AP_2(V_2 - w).$$

The solution is conveyed to the generator, and there heated until all the gas is

driven off. It then occupies the volume V_1 under the pressure P_1 and at the temperature T_1 .

The necessary quantity of heat Q'' is equal to the difference in quantities of internal heat, augmented by the exterior work accomplished. This work is equal to $P_1(V_1 - w)$ less the work of the pump, $(P_1 - P_2)w$.

We have then

$$Q'' = q_{e_1} - q_{e_2}' + mA U_1 - mA U_2' + AP_1(V_1 - w) - A(P_1 - P_2)w.$$

Adding this equation to the preceding, member to member, we find

$$Q'' = q_{e_1} - q_{e_2} + mA(U_1 - U_2) + AP_1V_1 - AP_2V_2.$$

This equation is established without taking account of the effect of exchange of temperature. There is furnished to the solution which enters the generator a quantity of heat precisely equal to $q_{e_1} - q_{e_2}$. The quantity of heat Q' to be supplied by the generator in order to bring the pressure of the gas from P_2 to

P_1 and from the temperature T_2 to T_1 is then

$$Q' = mA(U_1 - U_2) + AP_1V_1 - AP_2V_2. \quad (107)$$

Equations (101) and (105), which have been deduced for the case of compression by a mechanical force, give

$$AW_r = mA(U_1 - U_2) + AP_1V_1 - AP_2V_2,$$

which is identical with the preceding.*

We have then $Q' = AW_r$, provided that the temperature T_1 , in the case where the change of pressure of the gas is obtained by the heat combined with the chemical action, is the same as in the case where the change is due to a mechanical force. Experiment proves that it is nearly so.

It appears that the temperature to which it is necessary to heat the ammonia solution to obtain a given pressure is higher as the solution becomes

* It should be noted that this generalization enables the elevation of pressure undergone by the ammonia gas in the generator to be regarded as identical with the compression of the gas by a piston in a compression machine in which the gas is superheated by compression.

1

2

3

4

5

COMPARISON OF CALCULATED

$$T_2 = 437.4^{\circ}.$$

Pressure in Generator.		TEMPERATURE OF GENERATOR.	
		Observed	Calculated $T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{.24}$
Atm.	Lbs. per sq. in.	Deg. F.	Deg. F.
(1)	(2)	(3)	(4)
1½	22.05	118.4
2	29.40	136.4
2½	36.75	149.0
3	44.10	158.0
3½	51.45	167.0
4	58.80	176.0
4½	66.15	183.2
5	73.50	190.4
5½	80.85	197.6
6	88.20	201.2
6½	95.55	212.0
7½	106.58	222.8	222.8
7½	110.25	226.4	228.2

weak. In ice machines the solution conveyed to the generator contains rather less of the gas as the pressure in the refrigerating coils becomes more feeble. We understand therefore how the temperature T_1 ought to increase as the temperature T_2 , corresponding to the pressure in the refrigerating coils, diminishes. Unfortunately, precise experiments upon this point are wanting.

A series of observations made by M. Rouart upon a Carré machine is herewith given.

The first and second columns of each table give the absolute pressures in atmospheres and pounds per square inch; the third, the temperatures observed in the boiler; the fifth, the temperatures of water in the condenser; the sixth column gives the temperatures of the liquefied gas corresponding to the pressures in the first column (see Table V); the temperatures are those of the interior of the condenser, and are naturally more elevated than the exterior.

In the case of mechanical compression

TABLE IX.—COMPARISON OF CALCULATED AND OBSERVED TEMPERATURES OF AMMONIA IN GENERATOR OF ABSORPTION MACHINE.

SECOND SET OF OBSERVATIONS.

$T_2 = 487.4^\circ$; $P_2 = 2441$ lbs. per sq. ft., or 16.95 lbs. per sq. in.

Absolute Pressures.		TEMPERATURE OF GENERATOR.		Temperature of condenser water.	Temperature of interior of condenser corresponding to observed pressure	Difference of temperature between the inside and outside of condenser.
		Observed.	Calculated. $T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{.24}$			
Atmos.	Lbs. sq. in.	Deg. Fahr.	Deg. Fahr.	Deg. Fahr.	Deg. Fahr.	Deg. Fahr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
3	44.10	168.4	...	46.4
4½	66.15	194.0	...	46.4
5	73.50	201.2	46.4
5½	80.85	212.0	46.4
6	88.20	217.4	46.4
6½	95.55	222.8	...	46.4
7	102.90	230.0	...	48.2	57.4	...
7½	110.25	244.4	228.2	50.0	61.0	11.0
8	117.60	255.2	240.8	53.6	64.4	10.8
8½	124.95	266.0	249.8	58.1	68.0	9.9
9	132.30	276.8	260.6	59.0	71.1	12.1
9½	139.65	284.0	269.6	60.8	73.9	13.1
10	147.00	294.8	278.6	62.6	77.2	14.6
10½	158.02	296.6	293.0	66.2	81.3	15.1
11½	172.72	298.4	307.4	60.8?	86.7	25.9?
13	191.10	298.4	327.2	60.8?	91.0	30.2?
14	205.80	309.2	341.6	95.0	96.8	1.8
15	220.50	320.0	356.0	100.4	96.8	3.6
15½	227.85	325.4	356.0	100.4	104.0	8.6

the final temperature T_1 is related to the initial temperature and to the initial and final pressures as expressed by the equation (100), viz.:

$$T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{\frac{AB}{c_p}}.$$

The fourth column of the table gives the temperatures calculated by this formula, supposing $T_2 = 437.4$ and $P_2 = 2441$ lbs. per sq. ft. or 16.95 lbs. per sq. in.

For the mean pressures the calculated temperatures coincide nearly with the observations. For the higher pressures the calculated pressures are higher than the observed. But it is necessary to remark that in this case the watery vapor mixed with the gas exerts a greater influence, and that the true gas pressures ought to be sensibly less than the pressures which have served as a basis for calculation.

The condensation in the condenser and the evaporation in the cooler are brought about exactly as in the case

of the machines acting by mechanical force. We shall have then, as in § 29,

$$\begin{aligned} Q_1 &= mc_p(T_1 - T_1') + mr_1', \\ x_2 r_2 &= q_1' - q_2, \\ Q &= m(1 - x_2)r_2 = mr_2 - m(q_1' - q_2), \\ m &= \frac{V_2}{u_2 + \frac{1}{\delta}} = \frac{V_2}{v_2}, \\ Q' &= Q_1 - Q. \end{aligned}$$

The two following tables give the results of calculations for one cubic foot of ammonia gas, for temperatures in the condenser ranging from $+59^\circ$ to $+104^\circ$. In the first the temperature of the interior of the cooler is taken at $+5^\circ$. In the second table it is -22° .

The results indicated in Tables X and XI are high; they vary from 25,900 to 76,100 negative heat units for each pound of coal burned. We are far from attaining such results in practice.

We have omitted in our calculations to take into account two conditions which modify largely the theoretical results:

TABLE X.—PERFORMANCE OF AMMONIA ABSORPTION MACHINE NEGLECTING* THE CONSIDERATION OF THE HEAT WASTED IN MAINTAINING A CONSTANT TEMPERATURE IN THE ABSORBER.

FIRST CASE: $t_2 = 5^\circ$ Fahr., $m = .12061$.

Temperature corresponding to pressure of vapor in condenser $= t_1$	Temperature of generator $= t_1$	Heat units removed by condenser $= Q_1$	Negative heat units produced $= Q$	Heat units expended $= Q_1 - Q$	Work of compression.	Theoretical ratio of refrigerating effect and heat expended $\frac{Q}{Q_1 - Q}$	Refrigerating effect per pound of coal, assuming that each lb. imparts 10,000 B. T. U. to the generator.	Absolute pressure in generator $P_1 + 144$
Deg. F.	Deg. F.	B.T.U.	B.T.U.	B.T.U.			B.T.U.	Lbs. sq. in.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
59	154.6	71.81	63.47	8.34	7.61	76,100	105.99	
68	179.9	72.05	62.31	9.74	6.40	64,000	125.08	
77	205.1	72.26	61.13	11.13	5.49	54,900	146.64	
86	230.3	72.46	59.93	12.53	4.78	47,800	170.83	
95	255.4	72.61	58.70	13.91	4.22	42,200	197.88	
104	280.3	72.74	57.45	15.29	3.76	37,600	227.76	

* The figures for the ratio of the refrigerating effect to the heat expended are the same as for the conditions of the examples of a compression machine, Arts. 30 and 31, the object of the tables being apparently to illustrate the similarity between the two systems when the heat abstracted by the absorber is neglected (see foot-note, p. 119). Columns 1-5 are then identical for the two systems. The complete performance of the absorption type is shown in Tables XII-XV.

TABLE XI.—PERFORMANCE OF AMMONIA ABSORPTION MACHINE NEGLECTING THE CONSIDERATION OF THE HEAT WASTED IN MAINTAINING A CONSTANT TEMPERATURE IN THE ABSORBER.

SECOND CASE: $t_2 = -22^\circ$ Fahr., $m = .06386$.

Temperature interior of condenser $= t_1'$	Temperature of generator* $= t_1$	Heat units removed by condenser $= Q_1$	Negative heat units produced $= Q$	Heat units expended $= Q_1 - Q$	Theoretical ratio of refrigerating effect and heat expended $\frac{Q_1 - Q}{Q}$	Refrigerating effect per pound of coal, assuming that each lb. imparts 10,000 B. T. U. to the generator.	Absolute pressure in generator $P_1 \div 144$
Deg. F.	Deg. F.	B.T.U.	B.T.U.	B.T.U.		B.T.U.	Lbs. sq. in.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
59	224.1	40.28	32.93	7.35	4.48	44,800	105.99
68	252.2	40.50	32.31	8.19	3.95	39,500	125.08
77	280.2	40.70	31.69	9.01	3.52	35,200	146.64
86	308.3	40.90	31.05	9.85	3.15	31,500	170.83
95	336.2	41.07	30.41	10.66	2.85	28,500	197.83
104	364.0	41.23	29.75	11.48	2.59	25,900	227.76

* See foot-note, Table VII.

1st. The necessity of cooling the absorption chamber so that the solution of the gas may be readily accomplished.

2d. The influence of the water carried along with the gas.

We will now examine the influence of these two causes of loss.

§ 37. When ammonia gas dissolves in water considerable heat is disengaged.

MM. Fabre and Silbermann have measured this heat of solution and found it equal to 925.7 heat units for each pound of gas dissolved.

The liquid of the absorption chamber being employed continually in dissolving the gas from the refrigerating coils, rises rapidly in temperature, and as the solubility diminishes with the increased temperature, it soon reaches a condition at which it ceases to work. To insure successful working it is necessary, therefore, to cool the absorption chamber by means of a current of cold water, in such a manner as to maintain a constant temperature. We will suppose this temperature to be t_1'' .

If we denote by Q_1' the quantity of heat of which the absorption chamber is relieved, we shall evidently have

$$Q_1 + Q_1' = Q + Q'$$

or

$$Q' = Q_1 + Q_1' - Q.$$

On the other hand, the gas arriving at the condenser is always mixed with a certain quantity of steam, usually about 6 or 8 per cent. By employing a solution of calcium chloride instead of pure water for a solvent, the amount of watery vapor is reduced to about 3 per cent.

The presence of the steam reduces the efficiency to a notable extent. It carries off a portion of the heat of the generator, and, having arrived in the cooler, it does not evaporate, but, by holding a portion of the ammonia, prevents it from volatilizing. It impedes the action, then, nearly in the same way as the waste spaces in the mechanical action machines, but to a greater extent.

We will proceed to determine the influence of this introduction of water.

Let m , as before, be the weight of gas sent out from the generator; μ the weight of water accompanying it, and the quantities r and q affected by the index e shall relate to the water.

When the mixture passes into the condenser the steam becomes liquid and absorbs a certain weight $m'*$ of gas, and we have

$$m' = \frac{\beta_1' \mu}{\delta v_1'}, \quad (108)$$

β_1' being the number of cubic feet of gas soluble in one cubic foot of water whose temperature is t_1' ; $\frac{1}{v_1'}$ being the weight of a cubic foot of gas at this temperature, and δ being the weight of a cubic foot of water.

According to Carius, the coefficient of solubility of ammonia, a gas in water, is represented by the empirical formula

$$\dagger \beta = 1841.65 - 34.794t + .3663t^2 - .0016396t^3.$$

* The gas m' absorbed by the water is regarded as subject to the same laws of temperature as though it were free. See (107), § 36.

† This formula makes the solubility 0 for about 130° Fahr., but the Pontifex absorption machine is operated successfully with a temperature of 131° Fahr. in the absorber.

The quantity of heat Q_1 which will be absorbed by the condenser is equal to the quantity of heat necessary to lower the temperature of the weight m of gas from t_1 to t_1' , plus the quantity of heat necessary to liquefy the weight $m - m'$ of gas, plus the quantity of heat necessary to liquefy and lower to the temperature t_1' the weight μ of steam, plus the heat disengaged by the solution of the weight m' of gas.

We shall have then

$$Q_1 = mc_p(t_1 - t_1') + (m - m')r_1' + \mu(q_{e_1} - q'_{e_1} + r_{e_1}) + m's_1', \quad (109)$$

calling s_1' the heat disengaged by the solution of one pound of ammonia gas in water having the temperature t_1' .

On passing the mixture into the refrigerating coils, a certain quantity of the liquefied gas is volatilized until the pressure and temperature become equal, respectively, to P_2 and T_2 , the pressure and temperature of the refrigerant. The water will retain in solution a weight m'' of gas,

less* than m' , and given by the equation

$$m'' = \frac{\beta_2 \mu}{v_2 \delta}. \quad (110)$$

The quantity of gas volatilized ($m - m'$) x_2 is found by the equation †

$$\left. \begin{aligned} (m - m')x_2 r_2 + (m' - m'')s_2 \\ = (m - m')(q_1' - q_2) + \\ \mu(q_{e1}' - q_{e2}) + m'c_p(t_1' - t_2) \end{aligned} \right\}. \quad (111)$$

The quantity of refrigerating effect realized is

$$Q = (m - m')(1 - x_2)r_2, \quad (112)$$

or

$$\left. \begin{aligned} Q = (m - m')(r_2 - q_1' + q_2) + \\ (m' - m'')s_2 - \mu(q_{e1}' - q_{e2}) - \\ m'c_p(t_1' - t_2) \end{aligned} \right\}. \quad (113)$$

* Although the lower temperature would tend to absorb more gas in the water, the effect of the lower pressure preponderates to cause a disengagement, which makes m'' less than m' . The solubility is approximately directly as the pressure, according to the experiments of Roscoe, *Annales de Chimie*, 1859.

† This is the quantity volatilized by passing through a throttling cock, no external heat being applied. x_2 is the proportion that the weight of gas vaporized bears to the initial weight of anhydrous gas.

The quantity of heat Q_1' which it is necessary to abstract from the absorption chamber in order to maintain a constant temperature, is equal to the heat arising from the solution of $m - m''$ weight of gas, minus the heat necessary to raise the weight m of gas and the weight μ of water from t_2 to t_1'' .*

$$Q_1' = (m - m'')s_1' - mc_p(t_1'' - t_2) - \mu(q_{e1}'' - q_{e2}). \quad (114)$$

The quantity of heat Q' which it is necessary to employ at the boiler is equal to $Q_1 + Q_1' - Q$. We have then, applying the above values,

$$Q' = m[c_p(t_1 - t_1') + s_1'] + (m - m')[\lambda_1' - \lambda_2] + m'c_p(t_1' - t_2) - mc_p(t_1'' - t_2) + (m' - m'')(s_1' - s_2) + \mu(\lambda_{e1} - q_{e1}'').$$

The heat of solution s varies probably with the temperature and the pressure of the gas; but we do not know the law of this variation, and we therefore assume

* The weight of water μ was considered by Ledoux to be cooled to the temperature of the condenser t_1' . As in modern practice the temperature of the absorber is higher than that of the condenser, the symbol t_1'' has been introduced by the revisers.



Lowest temperature of the
33.67 lbs. absolute per square in.
of saturated ammonia gas at
.12061 lbs.

Temperature correspond- ing to pressure of vapor in condenser.	Absolute pressure of gen- erator.	Temperature of genera- tor.*
t_1'	$P + 144$	t_1
Deg. F.	Lbs. per sq. in.	Deg. F.
(1)	(2)	(3)
59	105.99	154.6
68	125.08	179.9
77	146.64	205.1
86	170.83	230.3
95	197.83	255.4
104	227.76	280.3

This table has been added to

In calculating Col. 8 it is
equivalent to an evaporation of
if the coal gives 10 per cent of
horse-power for a non-condens-
steam heats the feed-water to
per hour per horse-power of a
Engine Indicator.)

Columns 10 and 11 have been

* See foot-note, Table VII.

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Lowest temperature of the
pressure = 16.95 lbs. absolute per
cubic foot of saturated ammonia
generator = $m = .06386$ lbs.

	Temperature correspond- ing to pressure of vapor in condenser.	Absolute pressure of gen- erator.	Temperature of genera- tor.*	Q
	t_1'	P + 144	t_1	
	Deg. F.	Lbs. per sq. in.	Deg. F.	B
(1)		(2)	(3)	
59	The figures assume that the absorber has the same tempera- ture as the condenser.	105.99	224.1	
68		125.08	252.2	
77		146.64	280.2	
86		170.83	308.3	
95		197.83	336.2	
104		227.76	364.0	

* See foot-note, Table VII.

this quantity to be constant* and equal to 925.7 heat units per pound, as found by Favre and Silbermann, for ordinary temperatures and pressures.

Making $s_1' = s_2 = s$, $t_1'' = t_1'$, and $q_{e1}'' = q_{e1}'$ in the above equation, it becomes

$$Q' = m[c_p(t_1 - t_1') + s] + (m - m') \\ [\lambda_1' - \lambda_2 - c_p(t_1' - t_2)] + \mu(\lambda_{e1} - q_{e1}'); \quad (115)$$

we have further

$$m = \frac{V_2}{u_2 + \frac{1}{\delta}} = \frac{V_2}{v_2}.$$

§ 38. Tables XVI and XVII exhibit numerical results. The two cases of § 36 are taken, supposing that the weight of watery vapor carried over is 5 per cent of the weight of the gas circulating. [Tables XII and XIII have been added by the revisers to show the useful effect

* All experiments on the heat of solution have thus far been made at atmospheric pressure and with nearly pure water. Experiments made on a 35-ton Pontifex absorption machine, in which the weak liquor contained 12½ per cent by weight of ammonia and the strong liquor 25 per cent, confirm the probability of the hypothesis regarding the equality of $S_1' = S_2$. (See vol. x, Trans. Amer. Soc. Mech. Engrs., and Stevens Indicator, Jan. 1892.)

when no water is carried over, and Tables XIV and XV give comparative figures for compression machines.]

If we compare the figures of these tables with those of § 36, we find that the cooling of the absorption chamber and the presence of watery vapor reduce the useful effect to a considerable extent.

We see, further, that the useful effect diminishes if the temperature is lowered in the refrigerating coils, but that the results remain the same for the same temperature of the condenser.

In the machines employing mechanical power, the useful effect also diminishes if the temperature of the refrigerating coils is lowered.

§ 39. In the practical manufacture of artificial ice we estimate the performance at from 2160 to 2700 negative heat units for each pound of coal burned, which is about 65 per cent of the figures given in Tables XVI and XVII. The difference*

* Experiments made in 1890, *vide* Trans. Am. Soc. Mech. Engrs., vol. XII, page 326, in which the quantity of ammonia circulated was measured, show that a loss of upwards of 20 per cent is due to rarefying the

ECONOMY OF AMMONIA

Refrigerating effect of one cubic foot of .12061 lb. and taken into the compressor at this pressure and the correspond-

Temperature corresponding to pressure of vapor in condenser.	Absolute pressure in condenser.	Temperature at end of compression.*	Heat carried away from condenser.	Refrigerating effect in heat units.	Ratio of refrigerating effect to heat expended.	WATER.	
						Work of compression.	Per minute per ton of ice-melting capacity in 24 hours.
Deg. F.	Lbs. per sq. in.	Deg. F.	B. T. U.	B. T. U.		Ft.	Gals.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
59	105.99	154.6	71.81	63.47	7.61	6.	.89
68	125.08	179.9	72.05	62.31	6.40	7.	.92
77	146.64	205.1	72.26	61.13	5.49	8	.94
86	170.83	230.8	72.46	59.93	4.78	9	.96
95	197.83	255.4	72.61	58.70	4.22	10.	.98
104	227.76	280.3	72.74	57.45	3.76	11	1.00

* See foot-note, Table VII.

page 160.

ECONOMY OF AMMONIA

Refrigerating effect of one cubic foot or .06386 lbs! taken
to the compressor at this pressure and the corresp

Temperature correspond- ing to pressure of vapor in condenser.	Absolute pressure in con- denser.	Temperature at end of compression.*	Heat carried away from condenser.	Refrigerating effect in heat units.	Ratio of refrigerating ef- fect to heat expended.	Work of compression, Per minute per ton of ice-melting ca- pacity in 24 hours.
Deg. F.	Lbs. per sq. in.	Deg. F.	B. T. U.	B. T. U.		Ft. Gals.
(1)	(2)	(3)	(4)	(5)	(6)	(20)
59	105.99	224.1	40.28	32.93	4.48	5, .97
68	125.08	252.2	40.50	32.31	3.95	6, .99
77	146.64	280.2	40.70	31.69	3.52	6, 1.02
86	170.83	308.3	40.90	31.05	3.15	7, 1.04
95	197.88	336.2	41.07	30.41	2.85	8, 1.07
104	227.76	364.0	41.23	29.75	2.59	8, 1.09

* See foot-note, Table VII.

: 160.



ammonia. Refrigerating power
at the lowest temperature of the
retained in water carried into the
 $r = .0060805$ lb.

pound of coal.		Condensing water, assum- ing a total range of 60° for both condenser and absorption chambers.	
s.	r 142.2	Per ton of ice-melt- ing capacity. $\frac{Q_1 + Q_1' \times 142.2 \times 2000}{Q \times 60 \times 84}$	Per minute per ton of ice-melting ca- pacity in 24 hours. Col. 11 + 1440.
		Gals.	Gals.
(0)		(11)	(12)
8		1,870	1.30
6		1,890	1.31
4		1,920	1.33
2		1,960	1.36
0		2,010	1.40
10		2,090	1.45

To face page 160.

with the ammonia. Refrigerating
 s at the lowest temperature of the
 water carried into the refrigerating

pound of coal.	Condensing water, assum- ing a total range of 60° for both condenser and absorption chambers.	
	Per ton of ice-melt- ing capacity. $\frac{Q_1 + Q_1' \times 142.2 \times 2000}{Q \times 60 \times 8\frac{1}{2}}$	Per minute per ton of ice-melting ca- pacity in 24 hours. Col. 11 + 1440.
s.	Gals.	Gals.
0)	(11)	(12)
.2	1,900	1.32
.5	1,920	1.33
.8	1,950	1.35
.1	1,990	1.38
.1	2,050	1.42
.7	2,130	1.48

ux.

To face page 160.

tained without loss of useful effect, while in the ammonia machine -13° and -22° are readily and economically obtained.

We will not enter here upon questions of a purely practical character which affect the comparative values of the several ice machines, as our object has been simply to establish the theoretic conditions under which they work.*

* For a comparison of the difference between theory and practice in the various classes of machines, see foot-notes, pages 124 and 131, and the Introduction.

APPENDIX.

NOTE UPON THE DETERMINATION OF THE
LATENT HEAT OF VAPORIZATION, ALSO
OF THE SPECIFIC HEAT OF SULPHUR
DIOXIDE AND AMMONIA IN THE FORM
OF LIQUID. ALL VALUES IN BRITISH
UNITS.

It was shown in § 27 that the relation between the pressure, specific volume, and temperature of a liquefiable gas, being represented by the equation

$$Pv = BT - CP^n, \quad (116)$$

the constants B , C , and n can be determined by means of the coefficient of dilatation and the experiments of Regnault upon the compressibility of gases.

These constants are:

	For Sulphur Dioxide.	For Ammonia.
B	25.305,	95.31,
C	24.34,	157.11,*
n	.449,	.3783.

* See foot-note, p. 104.

Regnault determined also the elastic forces of these substances at different temperatures, and established the empirical formula

$$\log F = a + b\alpha^t + c\beta^t.$$

This form not being convenient for calculation, we have preferred to take the formula called Roche's,

$$\frac{P}{144} = a\alpha^{\frac{t-32}{1.8+m(t-32)}}, \quad (117)$$

and we have calculated the three constants a , α , and m for both sulphur dioxide and ammonia.

These constants are*

For Sulphur Dioxide.	For Ammonia.
$a = 22.535,$	61.847,
$\log a = 1.3528576,$	1.7913193,
$\alpha = 1.0415,$	1.03852,
$\log \alpha = .017657,$.0164175,
$l\alpha = .040657,$.037803,
$m = .004282,$.003998.

* These constants have been slightly altered by the revisers. See article in Stevens Indicator, Oct. 1890, in which the pressures for ammonia are made to correspond to Regnault's second table of pressures and temperatures. In the present case Regnault's first table of pressures and temperatures of ammonia is

Finally M. Regnault found for the specific heat of sulphur dioxide .15438, and of ammonia gas .50836.

On the other hand, Clausius established between the latent heat r , the absolute temperature T , the pressure P , and the quantity u , the relation

$$\frac{r}{u} = AT \frac{dP}{dt},$$

or

$$\frac{r}{APu} = \frac{T \frac{dP}{dt}}{P}; \quad (118)$$

u is the increase of volume of a unit of weight of a volatile liquid when transformed into vapor.

If v is the specific volume of the vapor, we have

$$u = v - \frac{1}{\delta},$$

employed, which gives more exact values than the second table for the range of temperature considered in the numerical examples.

δ being the density of the liquid in lbs. per cu. ft., and consequently

$$APu = ABT - ACP^n - \frac{AP}{\delta}. \quad (119)$$

The constants B, C, and n being known, the equation will give APu .

Knowing APu , we find r by eq. (118).

$$r = APu \frac{T}{P} \frac{dP}{dt}, \quad (119a)$$

or, in consequence of eq. (117),*

$$r = APu \cdot \frac{1.8T \cdot l\alpha}{[1.8 + m(t - 32)]^2}. \quad (120)$$

The equation (120) will give r in terms of T and APu .

Finally it was shown in § 27 that the quantity λ , that is, the total heat of vaporization, satisfies the equation

$$\lambda = \lambda_0 + c_p t - \frac{AC}{n} (P^n - P_0^n).$$

* From (117)

$$\begin{aligned} \frac{dP}{dt} &= 144 \times 1.8 \times \alpha \alpha^{\frac{t-32}{1.8+m(t-32)}} l\alpha + [1.8 + m(t-32)]^2 \\ &= \frac{1.8Pl\alpha}{[1.8 + m(t-32)]^2}. \end{aligned}$$

Substituting this relation in (119a) we obtain (120).

At the temperature of 32° we have

$$\lambda_0 = r_0,$$

then it becomes

$$\lambda = r_0 + c_p t - \frac{AC}{n}(P^n = P_0^n), \quad (121)$$

an equation in which P_0 represents the pressure of the vapor at 32° , c_p the specific heat of the vapor at constant pressure, and r_0 the latent heat at 32° .

The heat of the liquid

$$q = \lambda - r.$$

We shall have then

$$q = r_0 + c_p t - \frac{AC}{n}(P^n - P_0^n) - \frac{AP^n}{[1.8 + m(t - 32)]}, \quad (122)$$

and the specific heat of the liquid

$$c = \frac{dq}{dt}.$$

The equations (119), (120), (121), and (122), involving laborious calculations, we can replace the second member by

empirical expressions of the form $A' + B't + C't^2$, and then calculate the constants by means of three values taken at the two extremities and middle of the thermometric scale, and previously determined by aid of these equations.

We thus find for

SULPHUR DIOXIDE.

$$\begin{aligned} APu &= 14.144 + .02372t - .000064t^2, \\ r &= 171.971 - 0.2300t - .000095t^2, \\ \lambda &= 160.367 + .13188t - .000073t^2, \\ q &= -11.604 + .3619t + .000022t^2, \\ c &= .3619 + .0000444t. \end{aligned}$$

AMMONIA.*

$$\begin{aligned} APu &= 51.408 + .09071t - .000033t^2, \\ r &= 583.333 - .5499t - .001173t^2, \\ \lambda &= 552.188 + .3908t - .000157t^2, \\ q &= -31.145 + .9408t + .001016t^2, \\ c &= .9409 + .002032t. \end{aligned}$$

The specific heat of liquid ammonia is nearly equal to that of water. This re-

* These equations do not contain the correction for constant C explained in foot-note, p. 104, for the reasons given in the Preface.

sult, though astonishing at first, is comprehended when we reflect that the specific heat of the gas at constant pressure (.50836) is higher than that of steam (.4805). It would be interesting to verify by experiment the theoretical conclusion.*

The results obtained here for ammonia are, however, only approximate, for we need, in order to determine the constants of eq. (116), the coefficient of dilatation of this gas, and at present it is not known.

To facilitate calculations upon ice machines, we have prepared Tables XVIII and XIX for sulphur dioxide and ammonia. They give for each 9° the heat of the liquid q , the total heat of vaporization λ , the latent heat of vaporization r , the internal latent heat ρ , the external latent heat APu , and the weight of a

* This has been determined by Dr. Hans von Strombeck. See Journal of Franklin Institute, 1890, vol. ii. p. 471. The value found was 1.23 for the average specific heat between 85° and 140° Fahr. This is within 15 per cent of the figure given by Ledoux's table, and about the same as that given by Zeuner's table.

cubic foot of vapor $\frac{1}{v}$, for the temperatures between -40° and $+104^{\circ}$.

Zeuner* has recalculated Ledoux's table for ammonia, employing the corrected value of C. The results of his calculations are given in Table XXII.† Zeuner has also calculated a table for sulphur dioxide, basing the equations on the values determined by Cailletet and Mathias for the latent heat. (See Table XXI and the foot-note appended to the same.)

Table XX, of the properties of carbonic acid, is calculated by Prof. Schroter of Munich, and has been used in solving the numerical examples given in the text.

It will be observed that all of the general formulæ for equations (87) to (102) are based upon equation (86), which is known as Zeuner's formula for superheated gases. It was deduced by Zeuner many years ago, from the general equations of thermodynamics, by the aid of certain

* Added by the revisers.

† See foot-notes, p. 104 and at Table XXII.

approximate assumptions, and constants were determined from Hirn's experiments on superheated steam, which made the equation capable of representing the relation between pressure, volume and temperature of steam, either in the saturated or superheated condition.

The argument by which Zeuner established the formula, is given in full in the appendix to Rontgen's *Thermodynamics*, Du Bois's translation.

All attempts to deduce working formulæ for the properties of volatile vapors, such as formulæ 119 to 122 in the Appendix, are based upon Zeuner's equation, with the exception of those in Wood's *Thermodynamics*, which have been very ingeniously founded upon Rankine's general expression

$$pv = RT - a_0 - \frac{a_1}{T} - \text{etc.},$$

for relation between pressure and temperature, in which a_0 and a_1 are constants and the remainder of the notation is the same as that previously employed. See Wood's *Thermodynamics*, Art. 163.

TABLE XVIII.—LEDOUX'S TABLE FOR SATURATED SULPHUR DIOXIDE GAS.*
Heat units expressed in British Thermal Units per pound of sulphur dioxide.

Temperature of ebullition in deg. F. t	Absolute pressure in lbs. per sq. in. $P + 144$	Total heat reckoned from 32° Fahr. A	Heat of liquid reckoned from 32° Fahr. q	Latent heat of evaporation. r	Heat equivalent of external work. AP_u	Internal latent heat. p	Increase of volume during evaporation. u	Density of vapor or weight of one cubic ft.
Deg. Fahr.	Lbs.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	Cu. ft.	Lbs.
-22	5.56	157.43	-19.56	176.99	13.59	163.39	13.17	.076
-13	7.23	158.64	-16.30	174.95	13.83	161.12	10.27	.097
-4	9.27	159.84	-13.05	172.89	14.06	158.84	8.12	.123
5	11.76	161.03	-9.79	170.82	14.26	156.56	6.50	.153
14	14.74	162.20	-6.53	168.73	14.46	154.27	5.25	.190
23	18.31	163.36	-3.27	166.63	14.66	151.97	4.29	.232
32	22.53	164.51	0.00	164.51	14.84	149.68	3.54	.282
41	27.48	165.65	3.27	162.38	15.01	147.37	2.93	.340
50	33.25	166.78	6.55	160.23	15.17	145.06	2.45	.407
59	39.93	167.90	9.83	158.07	15.32	142.75	2.07	.483
68	47.61	168.99	13.11	155.89	15.46	140.43	1.75	.570
77	56.39	170.09	16.39	153.70	15.59	138.11	1.49	.669
86	66.36	171.17	19.69	151.49	15.71	135.78	1.27	.780
95	77.64	172.24	22.98	149.26	15.82	133.45	1.09	.906
104	90.31	173.30	26.28	147.02	15.91	131.11	.91	1.046

* In order to check the tables, curves have been plotted in which the ordinates represent the values given in the tables and the abscissae the temperatures. A few of the values have been altered slightly in order to make them afford a smooth curve.

TABLE XIX.—LEDoux's TABLE FOR SATURATED AMMONIA GAS.
Heat quantities expressed in British Thermal Units per pound of ammonia.

Temper- ature of ebullition t	Absolute pressure in lbs. per sq. in. $P + 144$	Total heat reckoned from 32° Fahr. λ	Heat of liquid reck- oned from 32° Fahr. q	Latent heat of evapora- tion. r	Internal latent heat. ρ	Heat equiv- alent of external work. AP_u	Increase of vol- ume dur- ing evap- oration. u	Density of vapor or weight of one cu. ft. $\frac{1}{v}$
Deg. Fahr.	Lbs.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	Cu. ft.	Lbs.
-40	10.22	536.30	-67.15	603.45	555.72	47.73	25.05	.040
-31	13.23	539.92	-59.33	599.25	550.69	48.56	19.70	.051
-22	16.95	543.51	-51.35	594.86	545.47	49.39	15.64	.064
-13	21.51	547.08	-43.20	590.29	540.07	50.22	12.53	.080
-4	27.04	550.62	-34.89	585.51	534.46	51.05	10.14	.098
5	33.67	554.14	-26.42	580.56	528.70	51.86	8.27	.121
14	41.58	557.63	-17.78	575.40	522.73	52.67	6.79	.147
23	50.91	561.09	-8.97	570.06	516.59	53.47	5.64	.177
32	61.85	564.53	0.00	564.53	510.25	54.28	4.71	.211
41	74.55	567.95	9.14	558.81	503.74	55.07	3.96	.251
50	89.21	571.35	18.43	552.92	497.06	55.86	3.36	.296
59	105.99	574.70	27.90	546.81	490.16	56.65	2.87	.346
68	125.03	578.04	37.53	540.51	483.08	57.43	2.47	.402
77	146.64	581.36	47.32	534.04	475.84	58.20	2.13	.465
86	170.83	584.64	57.28	527.36	468.40	58.96	1.86	.534
95	197.83	587.90	67.40	520.50	460.77	59.73	1.62	.610
104	227.76	591.14	77.69	513.46	452.97	60.49	1.43	.691

TABLE XX.—PROPERTIES OF SATURATED CARBONIC ACID GAS.

Heat units expressed in British Thermal Units per pound of carbonic acid.

[Transformed to English units from a Metric Table computed by Prof. SCHROTER.]

Temperature of ebullition in deg. F.	Absolute pressure in lbs. per sq. in.	Total heat reckoned from 32° Fahr.	Heat of liquid reckoned from 32° Fahr.	Latent heat of evaporation.	Heat equivalent of external work.	Internal latent heat.	Increase of volume during evaporation.	Density of vapor or weight of one cu. ft.
<i>t</i>	<i>P</i> + 144	λ	<i>q</i>	<i>r</i>	<i>APu</i>	ρ	<i>u</i>	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
-22	210	96.35	-87.80	136.15	16.20	119.95	.4138	2.321
-13	249	99.14	-82.51	131.65	16.04	115.61	.3459	2.759
-4	292	99.88	-26.91	126.79	15.80	110.99	.2901	3.265
5	342	100.58	-20.92	121.50	15.50	106.00	.2435	3.853
14	396	101.21	-14.49	115.70	15.08	100.62	.2042	4.535
23	457	101.81	-7.56	109.37	14.58	94.79	.1711	5.331
32	525	102.35	0.00	102.35	13.93	88.42	.1426	6.265
41	599	102.84	8.32	94.53	13.14	81.38	.1177	7.374
50	680	103.24	17.60	85.64	12.15	73.49	.0960	8.703
59	768	103.59	28.22	75.37	10.91	64.46	.0763	10.353
68	864	103.84	40.86	62.98	9.29	53.69	.0577	12.480
77	968	103.95	57.06	46.89	7.06	39.83	.0391	15.475
86	1080	103.72	84.44	19.28	2.95	16.33	.0147	21.519

This table has been added to those given by Ledoux.

TABLE XXI.—ZEUNER'S TABLE FOR SATURATED SULPHUR DIOXIDE GAS.
Heat units expressed in British Thermal Units per pound of sulphur dioxide.

Temperature of ebullition in deg. F. t	Absolute pressure in lbs. per sq. in. $P + 144$	Total heat reckoned from 32° Fahr. λ	Heat of liquid reckoned from 32° Fahr. q	Latent heat of evaporation. r	Heat equivalent of work. AP_u	Internal latent heat. ρ	Increase of volume during evaporation. u	Density of vapor or weight of one cubic ft.
Deg. F.	Lbs.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	Cu. ft.	Lbs.
-40	3.16	155.22	-17.76	173.98	12.30	160.68	20.89*	.049*
-31	4.23	156.39	-16.55	172.94	12.76	160.18	16.21	.063
-22	5.56	157.55	-15.05	172.60	13.18	159.42	12.71	.079
-13	7.23	158.69	-13.26	171.95	13.56	158.40	10.06	.099
-4	9.27	159.82	-11.18	171.00	13.89	157.11	8.04	.124
5	11.76	160.98	-8.82	169.75	14.18	155.56	6.47	.154
14	14.75	162.02	-6.17	168.19	14.43	153.75	5.25	.190
23	18.31	163.10	-3.23	166.33	14.64	151.68	4.29	.232
32	22.53	164.16	0.00	164.16	14.81	149.34	3.53*	.283*
41	27.48	165.21	3.52	161.69	14.94	146.75	2.92	.341
50	33.26	166.24	7.32	158.92	15.03	143.89	2.43	.410
59	39.93	167.25	11.41	155.84	15.07	140.77	2.02	.491
68	47.62	168.25	15.79	152.46	15.06	137.38	1.70	.584
77	56.39	169.23	20.45	148.78	15.06	133.72	1.43	.692
86	66.37	170.20	25.41	144.79	14.96	129.83	1.21	.819
95	77.64	171.15	30.65	140.50	14.84	125.66	1.08	.965
104	90.32	172.08	36.18	135.90	14.68	121.22	.87	1.131

This table has been added because it is made to agree with the experimental figures that have been determined for sulphur dioxide. Zeuner makes use of the work of Caillaet and Mathias who calculated the latent heat by employing the experimental values for the density of the gas at various pressures.

Empirical formulæ are made to agree with the results obtained for the latent heat of evaporation r and the external latent heat AP_u , and from these the remaining quantities are deduced. The results obtained in this way for the latent heat agree with those determined by direct experiments made at a later date by Mathias. See p. 182, Appendix.

* These values have been altered slightly so as to fit a smooth curve. See foot-note, Table XVIII.

TABLE XXII.—ZEUNER'S TABLE FOR SATURATED AMMONIA GAS.
Heat quantities expressed in British Thermal Units per pound of ammonia.

Temper- ature of ebulli- tion. <i>t</i>	Absolute pressure in lbs. per sq. in. $P \div 144$	Total heat reckoned from 32° Fahr. λ	Heat of liquid reckoned from 32° Fahr. q	Latent heat of evapora- tion. r	Internal latent heat. p	Heat equiva- lent of external work. APu.	Increase of volume during evapora- tion. u	Density of vapor or weight of one cu. ft. $\frac{1}{v}$
Deg. F.	Lbs.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	B. T. U.	Cu. ft.	Lbs.
-40	10.22	538.65	— 60.82*	589.47	550.69	48.77	25.61	.039
-31	13.23	542.34	— 54.54	596.88	547.33	49.55	20.10	.050
-22	16.95	545.96	— 47.88	593.87	543.53	50.32	15.93	.063
-13	21.51	549.54	— 40.84	590.38	539.32	51.06	12.74	.078
-4	27.04	553.08	— 33.43	586.51	534.72	51.78	10.28	.097
5	33.67	556.57	— 25.64	582.21	529.74	52.48	8.37	.119
14	41.58	560.00	— 17.47	577.47	524.32	53.16	6.86	.145
23	50.91	563.41	— 8.92	572.33	518.53	53.81	5.67	.175
32	61.85	566.75	0.00	566.75	512.30	54.45	4.73	.210
41	74.55	570.06	9.30	560.76	505.69	55.06	3.96	.251
50	89.21	573.31	18.98	554.33	498.67	55.65	3.35	.296
59	105.99	576.52	29.03	547.49	491.26	56.23	2.85	.348
68	125.08	579.68	39.46	540.22	483.44	56.78	2.44	.406
77	146.64	582.78	50.27	532.51	475.20	57.31	2.10	.471
86	170.83	585.84	61.45	524.39	466.58	57.81	1.82	.543
95	197.83	588.86	73.02	515.84	457.54	58.30	1.58	.622
104	227.76	591.83	84.95	506.88	448.11	58.77	1.39	.709

In this table the correct value of the constant C is employed in Zeuner's equation. See foot-note, p. 104. From a mathematical standpoint, this table is more exact than the one calculated by Ledoux. The possible error in the experimental value of the density of the ammonia vapor will, however, more than cover the difference in the results given in the two tables; and as the values in Ledoux's table are nearer to the figures deduced from Regnault's direct experiments on the latent heat of ammonia than those given by Zeuner, and also agree with the results obtained on a large scale by Prof. Deaton by metering the amount of ammonia circulated in a 75-ton ammonia-compression machine, we have employed Ledoux's table in all the numerical work given in the text. See p. 186, Appendix.

* This value has been altered slightly so as to fit as smooth curve. See foot-note, Table XVIII.

DISCUSSION OF THE EXPERIMENTAL DETERMINATIONS OF THE LATENT HEAT OF AMMONIA AND SULPHUR DIOXIDE.*

Experiments made by Regnault show that the heat required to evaporate ammonia at the temperature of 11.67°C ., or $53.01^{\circ}\text{Fahr.}$, and the corresponding pressure, and to reheat the gas to this temperature after it has been cooled by passing it through an orifice which reduces its pressure to that of the atmosphere, is 529.6 heat units per pound. This figure is greater than the latent heat of evaporation by the amount of heat required to reheat the gas. It is not possible to deduce the latent heat from the figure just given without employing theoretical values for some of the quantities involved. The probable latent heat deduced from Regnault's experiments at the above temperature and the corresponding absolute pressure of

* Abstracted mainly from matter presented at the Richmond meeting of the American Society of Mechanical Engineers (1890), and forming part of Volume XII. of the *Transactions*.

94.8 lbs. per sq. in., is 521.6 British thermal units per pound.

The only other laboratory experiments on the latent heat of ammonia that have been published are by Dr. Hans von Strombeck, chemist of the De La Vergne Refrigerating Machine Co.* The results in which he has the most confidence give 533.7 B. T. U. for the latent heat at the temperature of 63.84° Fahr. This figure falls between the value deduced from Regnault's experiments and that developed by Ledoux's theory.

The first experiment on the latent heat of sulphur dioxide, the results of which have been already given, were made by Favre and Silbermann in 1853.

The result obtained was 170.21 British heat units per pound, which agrees with the results obtained later by Chappuis and Mathias; the corresponding figure, calculated by means of an equation, found to agree with these experiments, being 172.28. The value found by Favre

* Journal of the Franklin Institute, Dec. 1890 and June 1891.

TABLE XXIII.

COMPARISON OF THE RESULTS DERIVED BY MEANS OF THEORETICAL FORMULÆ FOR THE LATENT HEAT OF AMMONIA, WITH THE RESULT FOUND BY REGNAULT.

Author-ity.	Date at which formula was furnished.	Formulæ for latent heat of evaporation in British Thermal Units per pound.	Temperature substituted in formulæ.		Latent Heat.		
			Cent.	Fahr.	Probable value deduced from Regnault's experiments.		Calculated.
					Calories per kilo-gramme.	British heat units per pound.	
Ledoux*.	1878	$583.3 - .5499t - .001173t^2$	11.67	53.01	289.8	521.6	306.0
"†.	"	$584.7 - .8777t + .000774t^2$	"	"	"	"	300.2
Peabody‡.	1889	$585.6 - .8t$	"	"	"	"	290.7
Wood§.	1889	$555.5 - .613t - .000219t^2$	"	"	"	"	290.2
Zeuner .	1890	$584.7 - .4754t - .002619t^2$	"	"	"	"	306.7
							550.8
							540.4
							528.2
							522.4
							552.1

The constants in the second formula by Ledoux have been altered so as to agree with the correct value of C in Zeuner's equation and the values of the constants in Roche's equation corresponding to Regnault's second table of pressures and temperatures of ammonia. In the above formulæ t is the temperature indicated by an ordinary thermometer scale, and is not the absolute temperature.

* *Annales des Mines*. July, 1878.

† *Stevens' Indicator*. Oct., 1880.

‡ *Thermodynamics of the Steam Engine*. By Prof. C. H. Peabody, p. 458.

§ *Transactions of the American Society of Mechanical Engineers*. Vol. x, 1889, p. 641.

|| *Zeuner's Thermodynamik*. Vol. II, p. 359.

in 1874 is lower than it should be according to the experiments of Chappuis, and those made later by Mathias. Only two experiments were made in obtaining this result. As the accuracy of the experiments depended on a constant rate of ebullition, it may be slightly in error on account of some variation of this rate, which, as is often remarked by Regnault, is extremely variable for fluids of high volatility. The result found by Chappuis, by means of an ice calorimeter, agrees with the results of the later experiments made by Mathias.

A comparison of the experimental with the calculated values of the latent heats is made in Tables XXIII to XXV. Fig. 7 represents graphically the experimental values of the latent heat of carbonic acid, and shows how closely the calculated values by Schroter are in agreement. The curve of experimental values in Fig. 7 is taken from Mathias's work on the latent heat of carbonic acid. *Annales de Chimie et de Physique*, 6th Series, vol. xxi. p. 128.

TABLE XXIV.

Authority.	Date at which experiments were published.	Conditions in regard to temperature and pressure under which the latent heat was determined.	Temperature of ebullition in degrees Fahr.	Latent heat of evaporation.	
				Calories per kilogramme.	British heat units per pound.
Favre and Silbermann *	1863	Atmospheric pressure...	13.86	94.56	170.21
Favre †	1874	"	13.86	92.22	147.99
Chappuis ‡	1888	32° Fahrenheit.....	32.00	91.70	165.06
Mathias §	1890	5.74° Centigrade.....	42.99	89.86	160.86
"	"	9.44° ".....	48.99	88.12	158.62
"	"	10.50° ".....	50.90	87.32	157.18
"	"	12.23° ".....	54.01	87.80	157.14
"	"	19.95° ".....	67.91	84.46	152.06

* *Annales de Chimie et de Physique*. 3d Series, vol. xxxvii, p. 470.

† *Ibid.* 5th Series, vol. i, p. 251.

‡ *Ibid.* 6th Series, vol. xv, p. 498.

§ *Ibid.* 6th Series, vol. xxi, p. 116.

TABLE XXV.
COMPARISON OF THE RESULTS DERIVED BY THEORETICAL FORMULÆ FOR THE
LATENT HEAT OF SULPHUR DIOXIDE WITH THE RESULTS FOUND BY EXPERIMENT.

Calculated Latent Heat. All equations in French units.																						
Tempera- ture substituted in formulæ.	Latent heat by ex- periment.	Ledoux, 1878,* $91.54 - .2339t$ $+ .0002357t^2$						Cailletet and Ma- thias, 1887, [†] $\frac{E}{T}(u' - u) \frac{dp}{dt}$			Peabody, 1889, [‡] $98 - .27t$			Mathias, 1890, [§] $91.87 - .384t$ $- .000340t^2$			Mathias, 1890, $91.87 - .384t$			Zeuner, [¶] 1890, $91.2 - .2576t$ $- .000375t^2$		
		Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.	Calories per kilo.	B. T. U. per pound.			
-10.08	18.86	94.56	170.21	93.7	168.66	100.72	181.30	95.71	172.98	95.74	172.88	93.45	168.21									
0	32.	91.70	165.06	91.2	164.16	98.00	176.42	91.87	165.37	91.87	165.37	91.30	164.16									
5.74	42.33	89.38	160.88	89.2	160.56	96.45	173.61	89.66	161.39	89.67	161.41	89.61	161.30									
9.44	48.99	88.12	158.62	88.8	159.84	95.45	171.81	88.92	158.79	88.25	158.85	88.47	159.25									
10.50	50.90	87.82	157.18	88.5	159.30	95.17	171.31	87.80	158.04	87.84	158.11	88.12	158.62									
12.23	54.01	87.30	157.14	87.8	158.04	94.70	170.46	87.12	156.92	87.17	156.91	87.54	157.57									
19.05	67.91	84.48	152.06	84.7	152.46	92.62	166.70	84.07	151.88	84.21	151.58	84.72	152.50									

The experimental figure at -10.08° is that determined by Favre and Silbermann, and that at 0° by Chappuis.

The remainder are those determined by Mathias.

The temperatures indicated by t are those indicated by a Centigrade thermometric scale, and are not absolute temperatures.

* *Annales des Mines*. July, 1878.

† *Journal de Physique*. 1887, pp. 423 and 424. In this formula

$$T = t + 273;$$

E = the mechanical equivalent of heat = 425;

w' = the specific volume of vapor at t degrees;

u = specific volume of liquid at t degrees;

p = pressure at temperature t .

The results given above, as determined by this formulæ, are found by interpolating in a table the numerical values given by Cailliet and Mathias.

‡ *Thermodynamics of the Steam Engine*. By Prof. C. H. Peabody.

§ *Annales de Chimie et de Physique*. 6th Series, vol. **xxi**, p. 117.

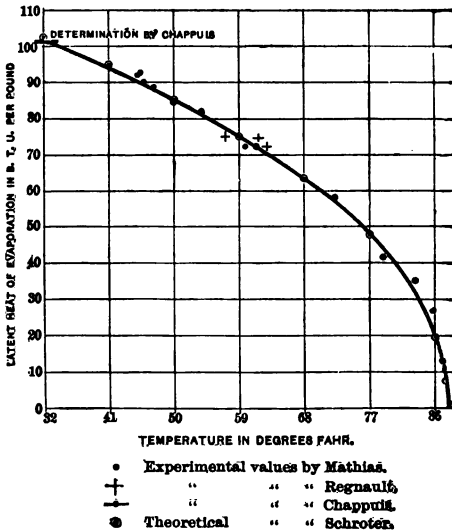
|| *Ibid.* 6th Series, vol. **xxi**, p. 118.

¶ *Technische Thermodynamik*. By Gustav Zeuner. Vol. **ii**, p. 264.

Experiments made by Prof. Denton
by metering the ammonia circulated in

Fig. 7.

LATENT HEAT OF CARBONIC ACID.



a compression machine of 75 tons capacity, gave the following results for the

latent heat at the suction and condenser pressures :*

Temperature corresponding to suction pressure, deg. Fahr.....	-10.7	-3.2	+14.5
Latent heat in { By Experiment....	569.6	603.5	570.4
B. T. U. per lb. { By Ledoux's table	589.1	585.1	575.1
Temperature corresponding to condenser pressure.....	84.6	82.7	87.7
Latent heat in { By Experiment....	524.8	525.7	512.4
B. T. U. per lb. { By Ledoux's table	528.4	529.8	526.1

These figures indicate that Ledoux's table is correct within the error of the experiments, which are trustworthy to within about 5 per cent.

* See paper presented at San Francisco meeting of Am. Soc. of Mechanical Engineers, May 1892.

ADDENDUM.

REFRIGERATING MACHINES USING VAPOR OF WATER.

IN these machines, sometimes called vacuum machines, water, at ordinary temperatures, is injected into, or placed in connection with, a chamber in which a strong vacuum is maintained. A portion of the water vaporizes, the heat to cause the vaporization being supplied from the water not vaporized, so that the latter is chilled or frozen to ice. If brine is used instead of pure water, its temperature may be reduced below the freezing-point of water. The water vapor is compressed from, say, a pressure of one-tenth of a lb. per square inch to one and one-half lbs., and discharged into a condenser. It is then condensed and removed by means of an ordinary air-pump. The principle of action of such a machine is the same as that given for

volatile vapor machines (section 29). The following is a numerical example:

For steam, the constants in Zeuner's equation are:

$$B = 92.84, \quad C = 938.9, \quad n = 0.25.$$

The specific heat at constant pressure, as determined by Regnault, is:

$$C_p = 0.475.$$

Hence
$$\frac{AB}{C_p} = 0.253.$$

Assume that ice is made, and that the lower temperature is 32 degrees Fahr. Let the pressure in the condenser be $1\frac{1}{2}$ lbs. per square inch, then we have

$$\frac{P_2}{144} = 0.089 \text{ lb. per sq. in.}; \quad t_1' = 116^\circ \text{ F.,}$$

and

$$T_1 = T_2 \left(\frac{P_1}{P_2} \right)^{0.253} = 1003.9 \text{ or } t_1 = 544.5^\circ \text{ F.}$$

The weight of one cubic foot of steam at 32 degrees is .000310 lbs.; hence

$$m = 0.00031.$$

By equation 103 we have

$$Q_1 = 0.3833 \text{ B. T. U.}$$

Equation 104 gives

$$Q = 0.3265 \text{ B. T. U.}$$

$$AW_r = Q_1 - Q = 0.0568 \text{ B. T. U.}$$

and

$$W_r = 43.85 \text{ ft. lbs.}$$

The useful effect, neglecting friction, is .00745 B. T. U. per foot-pound of work expended, not including friction. The ice-melting capacity in tons per cubic foot of piston displacement is .00000115, or the volume of the compressing cylinder is about 150 times the theoretical volume for an ammonia machine working under ice-making conditions, as per line 2, col. 16, Table VII.

The number of gallons of condensing water per ton of ice-melting capacity, assuming a range of 30 degrees Fahr. in the condensing water, is 1340. The ice-melting capacity per lb. of coal, assuming 3 lbs. of coal per hour per horse-power,

is 34.5 lbs., neglecting friction. Ammonia for ice-making conditions gives 40.9 lbs. (Table VII.).

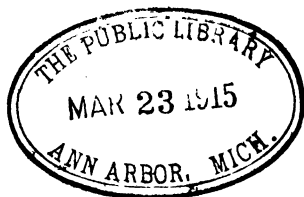
No calculation can be made including friction, as there is no experimental data available in regard to the friction losses in this class of machine.* The power to drive the air-pump employed to preserve the vacuum of $1\frac{1}{2}$ pounds in the condenser is also not included in the analysis, so that in all probability the actual power would exceed the theoretical [W_r] by about ten per cent.

Another type of machine, using vapor of water, absorbs the vapor in sulphuric acid, which is injected into the vacuum chamber and pumped out of the latter against a low pressure maintained in a still or concentrator, where the water in the acid is driven from it by heat, and from whence it is returned to the vacuum chamber. The principle of the action of this type of machine is similar to that

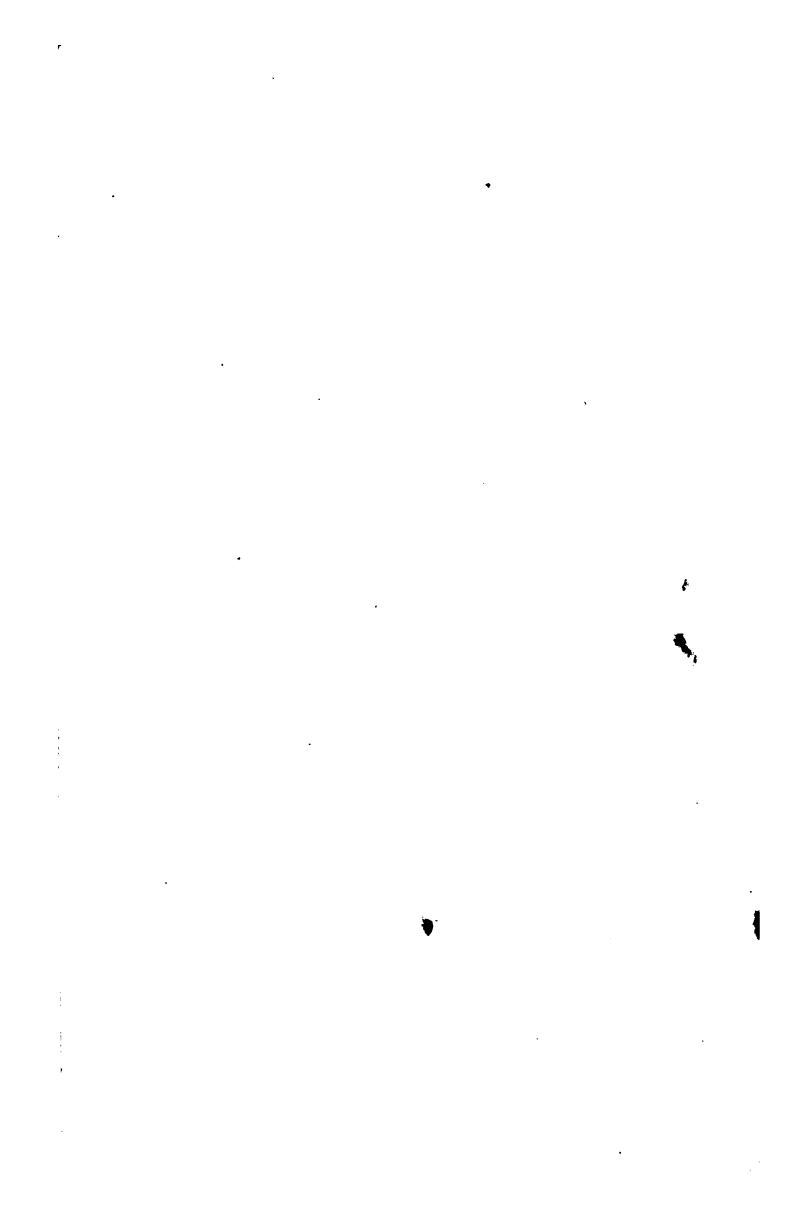
* A machine recently constructed on this principle had a compression cylinder 20 feet in diameter and 10 feet high, moved at 20 revolutions per minute.

of an ammonia absorption machine, the volume of the circulating pump being much less than that of the compressing cylinder for the case, where no absorbent is used for the water vapor. The principal expense of power is the heat to dissociate and evaporate the water from the acid. A theory of the action of this type of machine is reserved for the second volume of this work.

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